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Incorporating uncertainty and nonlinearity into the calculus of an efficient response to the threat of global warming

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Abstract: The possible serious social and economic consequences of global warming raise a series of questions. Some researchers have focused on the potential damage associated with global change; others have considered how such damage could be mitigated. Still others now investigate the range of uncertainty with which the future can be viewed. The fundamental policy and research issue straddles all of these more focused questions and calls for an integrated and dynamic analysis. This paper extends the Nordhaus framework, introduced in 1991, to address two omissions: the effect of cascading uncertainty across a variety of sources and the potential for severe nonlinearities in damages. The results suggest not only that more aggressive emission reduction strategies are in order, but also that it is appropriate for researchers to focus on emissions and potential damage trajectories which lie either above the mean or above the most likely scenario.

Key words: climate change, climate modelling, economics, emissions control, environmental damage, environmental damage mitigation, Nordhaus climate change model.

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* Throughout this article numbers in square brackets denote references, while superior numbers refer to notes listed at the end.

1 INTRODUCTION

The possibility that greenhouse-induced global change will have serious social and economic consequences has led a growing number of economists and other social scientists to confront a long series of questions – questions raised by the social and economic risks associated with those consequences. Some have begun to focus their attention on the damage which might be associated with global change; the US Environmental Protection Agency has, for example, undertaken a series of impact studies starting first with a domestic perspective but now expanding their scope more globally.[1] Others have turned to ponder the types of adaptive responses which would, by virtue of government action or private reaction, serve to mitigate that damage.[2] Still others now investigate the range of uncertainty with which we view the future and wonder how distributions of possible futures fit into the quantification of damage estimates and provide insight into which scenarios to analyse most closely.[3] Perhaps most of the collective effort of the research community has been devoted to assessing the cost of meeting certain mitigation targets.[4] The fundamental policy and research issue – the characterization of an efficient abatement strategy which appropriately weighs its cost against the potential damage of global change given enormous uncertainty about what the future might hold – straddles all of these more focused questions, of course, and calls for an integrated and dynamic analysis.

Nordhaus published the first attempt at such an integrated analysis in 1991. He viewed the problem as one of dynamic optimization with the costs of abatement and the potential damage of global change incorporated explicitly in a global objective function. His approach was, however, deterministic in that it worked with one baseline trajectory of future economic activity and associated emissions, one well defined marginal cost schedule for reducing carbon emissions based on a regression analysis cost estimate, and one marginal damage estimate based upon an EPA [2] analysis of the vulnerability of the United States to global warming. The model, and thus its result that only modest effort to reduce carbon emissions is warranted on efficiency grounds, therefore ignored cascading uncertainty across a variety of sources and the potential for severe nonlinearities in damages.

The present paper will extend the Nordhaus framework to incorporate both of these omissions. It will, in particular, look to expand the dynamic optimality condition which weighs discounted marginal cost and marginal damage estimates so that it weighs, instead, discounted expected marginal cost and marginal damage estimates.[5] Section 2 summarizes the basic Nordhaus model, focusing only upon its analytical structure. Section 3 builds uncertainty and nonlinearity into the damage side of the efficiency condition; the efficient response is seen to escalate even with a risk neutral objective function. Section 4 uses probabilistic scenario analysis of a simplified version of the 1983 global carbon emissions model of Nordhaus and Yohe [13] to examine the possibility that uncertainty and nonlinearity on the cost side might diminish the escalation suggested in Section 3; very little reason is found to accept such a hypothesis. Section 5 subsequently notes that the emissions scenarios produced in the analysis of Section 4 add a dimension of variable pace to the original Nordhaus baseline trajectory of emissions. The net effect of pace is small along the main trajectory, but is seen to dramatically increase the efficient emissions reduction target along possible and not unlikely alternative emissions trajectories. Concluding remarks in Section 6 finally highlight the cumulative effect of the extensions developed in earlier sections. They suggest not only that more aggressive emission reduction strategies are in order, but also that it is appropriate for researchers to focus on emissions and potential damage trajectories which lie either above the mean (of some probabilistic analysis) or above the most likely scenario.

2 THE NORDHAUS MODEL: COMPUTING THE EFFICIENT MITIGATION RESPONSE[6]

The operative model, developed by Nordhaus [11] and applied here, begins with a simplified temperature adjustment process characterized mathematically by:

$$(\mathrm{d}T/\mathrm{d}T) = a\{\mu M(t) - T(t)\} \text{ with,} \tag{1}$$

$$(\mathrm{d}M/\mathrm{d}t) = be(t) - \delta M(t) \tag{2}$$

Notationally, the variables $T(t)$, $M(t)$, and $E(t)$ represent the driving forces behind potential global environmental change. More specifically,

1 $T(t)$ represents the increase in global mean temperature through time t generated by greenhouse warming since the preindustrial period of the middle of the last century;

2 $M(t)$ represents the atmospheric concentration of greenhouse gases at time t denominated in terms of carbon dioxide equivalents; and

3 $E(t)$ represents the emission in time t of greenhouse gases, again denominated in terms of carbon dioxide equivalents.

Parameters a, μ, b and δ meanwhile define the relationships, with

1 a reflecting a delay parameter which correlates a realized increase in temperature to a prior increase in radiative forcing;

2 b indicating the fraction of carbon equivalent emissions which actually remain airborne;

3 δ representing a corresponding physical decay parameter for aggregated atmospheric concentrations of greenhouse gases; and

4 μ representing the (linearized) sensitivity of equilibrium temperature change to changes in atmospheric concentrations of greenhouse gases.

The economic side of the model is, meanwhile, summarized by

$$c(t) = y(t)\{g(E*) - \phi(T*)\} \text{with} \tag{3}$$

$$y(t) = y*e^{ht} \tag{4}$$

Notationally,

1 $c(t)$ represents per capital consumption at time t;

2 $y(t)$ represents per capital output growing in the absence of any emissions reduction and any deleterious effects of climate change at an annual rate of h;

3 $g(E*)$ represents a steady state computation of the cost of reducing emissions of greenhouse gases; and

4 $\phi(T*)$ represents a steady state computation of the economic damage associated with climate change.

Temperature is used as an index of climate change. Both the potential cost of climate change and the cost of mitigation are measured in terms of the long run equilibrium consistent with any change in radiative forcing.

A risk-neutral variant of the Nordhaus model was explored as an exercise in long run optimization with a linear objective function:[7]

$$W = \int \{c[t]\} e^{-\beta t} dt \qquad (5)$$

The condition which characterizes the solution of the long term optimization problem – maximize W subject to the constraints imposed on the system by equations (1) through (4) – states quite simply that the present value of any small change in the emissions trajectory should be zero; i.e. the immediate increase in per capita consumption associated with a small increase in emissions should be matched by an increase in the present value of the damage, denominated in reduced consumption, association with the long run effect of those higher emissions.[8]

Nordhaus showed that this simple statement of the optimality condition amounts to requiring that:

$$y * g'(E *) dE = \int \left[y * e^{ht} \phi'(T *) dT \right] e^{-rt} dt \qquad (6a)$$

Equations (1) and (2) meanwhile combine under the assumption that $\delta << a$ to define $dT(t)$ in terms of physical parameters; more specifically,

$$dT(t) == \mu b e^{-\delta t} \left[1 - e^{-at}\right] dE \qquad (6b)$$

As a result, equation (6a) simplifies to

$$g'(E*) = \mu b \phi'(T*) A \qquad (6c)$$

where the last term, A, is given by

$$A = \frac{1}{r - h + \delta} - \frac{1}{r - h + \delta + a} \qquad (6d)$$

Equation 6(c) is sufficient to support estimates of efficient responses to the threat of greenhouse warming. Recognizing uncertainties and nonlinearities on both sides of (6c) therefore held the promise of revealing the degree to which that efficient response should be adjusted. An amended optimality condition, that the immediate increase in consumption associated with a small increase in emissions should be set equal to the present value of damage associated with the long run effect of those emissions along any scenario, could be used to produce a distribution of efficient responses contingent upon those scenarios. Aggregating over that range of responses then produced a second amended optimality condition, that the immediate increase in consumption associated with a small increase in emissions should be set equal to the expected present value of damage associated with the long run effect of those emissions.

3 UNCERTAINTY ON DAMAGE SIDE [9]

The marginal damage of emissions is the primary economic component of the right-hand side of equation (6c). Nordhaus produced his point estimate of this component by relating damage statistics offered by the US Environmental Protection Agency for a baseline warming scenario to the most vulnerable sectors of the national income accounts of the United States. He assumed, in creating his estimate, that the most likely scenario would see an effective doubling of carbon dioxide concentration by the middle of the next century. Generating a series of marginal damage estimates across a range of possible futures requires more than a single baseline estimate, however; it requires, instead, a marginal damage schedule defined throughout that range. Such schedules are few and far between, but one does exist for the economic vulnerability of the United States to greenhouse induced sea level rise.

Estimates of national vulnerability for the United States expressed as a function of greenhouse induced sea level rise were employed to compute a marginal damage function of the form:

$$\phi'(SLR) = g \phi_0 e^{gSLR} \qquad (7)$$

with $g = 0.0253$ for $SLR < 80$ cm and $g = 0.0109$ for $SLR > 80$ cm.[10] Distributions of future sea level rise related to anticipated increases in equilibrium temperature were also required. A density function

$$f(SLR) = \alpha e^{-\alpha(SLR)} \qquad (8)$$

was taken with a mean $[=(1/\alpha)]$ dependent upon an assumed doubling sensitivity for temperature.[11] The IPCC Scientific Assessment meanwhile offers a best guess that an effective doubling of carbon dioxide concentrations would force a 2.5°C increase in equilibrium temperature and a 66 cm increase in sea level rise by the year 2100. The low end of the temperature range reported there stands at 1.5°C, presumably associated with the low end of the reported potential for sea level rise (33 cm through 2100); the high end of temperature sensitivity stands at 4.5°C with a 99 cm sea level rise. Using T_o = 2.5°C as the basis for a temperature index, $[T_d / T_o]$, and taking the IPCC sea level scenarios as mean estimates, a two-part linear relationship between the temperature index and the α required in underlying sea level density function was quantified:

$$\alpha(T_d) = 0.040 - 0.04[T_d / T_o] \quad [T_d / T_o] < 1$$
$$\alpha(T_d) = 0.023 - 0.007[T_d / T_o] \quad [T_d / T_o] \geq 1. \tag{9}$$

Letting SLR_o represent the expected sea level rise associated in equilibrium with the IPCC best guess doubling temperature sensitivity of 2.5°C, a short Taylor expansion of the right-hand side of equation (7) combined with equations (8) and (9) to produce

$$E\{\phi'(T_d)\} = \phi_o g SLR_o e^{gSLR_o} \int e^{g(SLR(T_d) - SLR_o)} f(SLR / T_d) dSLR \tag{10a}$$

for any given T_d with

$$f(SLR / T_d) = \alpha(T_d) = \alpha(T_d) e^{-\alpha(T_d)}.$$

As a result,

$$E\{\phi'(T_d)\} = \phi_o g SLR_o e^{gSLR_o} \frac{\alpha(T_d)}{[\alpha(T_d) - g]} e^{-gSLR_o} \tag{10b}$$

The other terms on the right-hand side of equation (6c) were less troublesome. Looking first to μ (the sensitivity of increase in equilibrium temperature to a change in the concentration of greenhouse gases), notice that $[dT(t)/dt] = 0$ when equilibrium has been achieved, so equation (1) reduces to $T^*(t) = \mu M^*(t)$ and

$$\mu \approx \{T_d / [M(0) \ln(2)]\} = \{T_o / [M(0) \ln(2)]\} \{T_d / T_o\}.$$

It became essential, therefore, to explore the subjective distribution of the previously defined index of doubling temperature sensitivity; i.e., the distribution of $\{T_d / T_o\}$. The IPCC Scientific Assessment [4] provides some insight into that distribution, but not very much. Its authors weighed the evidence from a series of recent studies against modelling results provided to the IPCC from independent researchers to conclude that:

> ...the sensitivity of global mean surface temperature to doubling (effective) carbon dioxide (concentrations) is unlikely to lie outside the range of 1.5 to 4.5°C. There is no compelling evidence to suggest in what part of this range the correct value is most likely to lie. There is no particular virtue in choosing the middle of the range, and both the sensitivity and the observational evidence neglecting factors other than the greenhouse effect indicate that a value in the lower part of the range may be more likely. Most scientists decline to give a single number, but for the purpose of illustrating IPCC scenarios, a value of 2.5°C is considered to be the 'best guess' in the light of current knowledge. [p. 139].

Setting 2.5°C as the benchmark 'best guess' T_o, this passage suggested that the doubling temperature increase index $\{T_d / T_o\}$ could be as low as 0.6 or as high as 1.8. Placing equal weight on the likelihood that 0.6, 1.2, and 1.8 will turn out to be the correct value (to reflect the 'no compelling evidence to suggest in what part of this range the correct value is most likely to lie' phrase) yielded 0.24 as an estimate of an implicit variance for a representative subjective distribution. Imposing a gamma distribution over the range to capture the notion that the 'best guess' index number is 1 (for a doubling temperature of 2.5°C) then suggested that $f_{T_d}\{T_d\} \equiv \Gamma(5;6)$ could be offered as a reasonable density function.[12]

The discount parameter A depends, for the most part, upon economic parameters which are best handled with sensitivity analysis[13] Parameters b, r, h and δ represent either the expected values of underlying random variables which are uncorrelated with anticipated doubling sensitivities or specific values for exogenous economic variables which frame the overall growth context of the greenhouse problem. The expected value of the right-hand side of equation (6) was thus characterized fully by:

$$E\{\text{RHS}\} = Ab \frac{T_o}{\ln\{2\}} \int [T_d / T_o] E\{\phi'(T_d)\} f_{T_d}[T_d] dT_d$$
$$= Ab \frac{T_o}{\ln\{2\}} \phi_o g SLR_o e^{gSLR_o} \int [T_d / T_o] \frac{\alpha(T_d)}{[\alpha(T_d) - g]} e^{-gSLR_o} f_{T_d}[T_d] dT_d \tag{11a}$$

Every term to the left of the integral sign is captured in the Nordhaus baseline estimate of marginal damage. Everything to the right, therefore, is part of an uncertainty index which can exaggerate or diminish the original baseline statistic. The key to producing an understanding of the degree to which uncertainty would cause the expected marginal damage of increased emissions to exceed the baseline estimate therefore lay in understanding the degree to which this uncertainty index,

$$\pi = \int [T_d / T_o] \frac{\alpha(T_d)}{[\alpha(T_d) - g]} - e^{-gSLR_o} f_{Td}[T_d] dT_d$$
$$\equiv \int [T_d / T_o] D[T_d] f_{T_d}[T_d] dT_d \tag{11b}$$
$$\equiv \int \pi[T_d / T_o] f_{T_d}(T_d) dT_d,$$

exceeds unity. Producing a range of possible marginal damage statistics contingent upon specific temperature sensitivities within the quoted IPCC range therefore lay ultimately in investigating the range of values which might be assumed by the various $\pi[T_d/T_o]$ – the weighted marginal damage multipliers defined implicitly by equation (11b).

Table 1 displays the critical results, given the specifics of the modelling extension described above. The second column records the marginal damage multiplier for each $[T_d/T_o]$ index listed – the $D[T_d]$ parameter implicitly defined in equation (11b) as

$$D[T_d] = \frac{\alpha(T_d)}{[\alpha(T_d) - g]} e^{-gSLR_o}.$$

Table 1 *The uncertainty multiplier, marginal damages, and efficient reductions in carbon emissions.*

(1)[c] T_d/T_o	(2)[d] $D[T_d]$	(3)[e] $\pi[T_d/T_o]$	(4)[a] *Marginal damage* $	(5)[b] *Efficient reduction* %
0.6	0.76	0.45	5.84	2.8
0.7	0.86	0.60	7.62	3.7
0.8	1.03	0.82	10.41	5.0
0.9	1.37	1.28	16.26	7.6
1.0	1.42	1.42	18.03	8.5
1.1	1.53	1.68	21.34	9.9
1.2	1.67	2.01	25.53	11.7
1.3	1.86	2.42	30.73	14.0
1.4	2.12	2.97	37.72	16.9
1.5	2.52	3.77	47.88	20.9
1.6	3.17	5.07	64.39	27.1
1.7	4.47	7.61	96.65	37.7
1.8	8.33	15.00	190.50	60.7

Notes:

(a) Computed as the product of complete uncertainty multiplier and the middle Nordhaus [11] marginal damage estimate of $12.70.
(b) Computed by comparing the marginal damage statistic computed in Column (4) with the marginal cost of emissions reduction estimated by Nordhaus from published long run energy analyses by regressing cost against the logarithm of one minus the percentage reduction; see footnote 10 and Figure 5, Nordhaus [11].
(c) An index of the increase in equilibrium global mean temperature associated with an effective doubling of atmospheric concentrations of carbon dioxide with 1 = 2.5°C.
(d) The expected marginal damage multiplier computed as instructed in equation (11b) according to the marginal cost structure defined by equation (7b) and the distribution of sea level rise given by equations J(8) and (9) for the doubling sensitivity specified in Column (1).
(e) The complete marginal damage multiplier defined by equation (11a).

Column (3) records the corresponding weighted marginal damage multiplier – the $\pi[T_d/T_o]$ computed according to equation (11b) as the product of the index value of Column (1) and the damage multiplier shown in Column (2).

Notice that these uncertainty multipliers run from a low of 0.45, for a doubling temperature index of 0.6 (doubling associated with 1.5°C), to a high of 14.99 for a temperature index of 1.8 (an equilibrium doubling temperature of 4.5°C).

These values represent the expected damage multiplier contingent upon the indicated value of $[T_d/T_o]$. Given $[T_d/T_o]$ = 0.6, for example, T_d =1.5°C and $\pi[T_d/T_o]$ = 0.46 so that the expected marginal damage estimate given a doubling temperature of 1.5°C is $5.84.[14] For $[T_d/T_o]$= 1.0, T_d= 2.5°C, $\pi[T_d/T_o]$ = 1.42 is the resulting expected damage multiplier, and $18.03 (US) represents the contingent estimate of expected marginal damage. Notice that uncertainty in our understanding of possible sea level rise trajectories even given the best guess temperature estimate increases marginal damage by 42%. On the opposite extreme, the contingent estimate of expected marginal damage is $190.50 when T_d = 4.5°C so that $[T_d/T_o]$ = 1.8 and the expected damage multiplier is 15.00. The expected value calculation prescribed by equation (11b) yields a mean of 2.64 over the entire range – a value roughly matching the 70th percentile of the $\pi[T_d/T_o]$ distribution.

Recall that the multipliers of Column (3) are translated into marginal damage estimates in Column (4) by multiplying them by the middle value reported by Nordhaus (i.e. $12.70). A mean of $33.53 lies above the median of a distribution stretching from $5.84 on the low end to $190.37 on the high side. Column (5) finally relates these marginal damage statistics to the marginal cost of reducing carbon emissions, thereby suggesting a range of emission reductions which could prove to be efficient.[15] Emissions reductions supported by an efficiency criteria which equates their marginal cost with the expected marginal damage of allowed emissions run from a 3% reduction in cumulative emissions through 2050 (if a 1.5°C increase in the global mean temperature were associated with an effective doubling of carbon concentrations) up to a 61% reduction in cumulative emissions (if doubling were to cause a 4.5°C increase). The mean percentage reduction, roughly equal to 14%, lies slightly below the 15% reduction supported by the mean damage multiplier.

The specific estimates recorded in Table 1 are certainly the product of the underlying structure. They are, however, quite insensitive to changes in the distribution of the doubling temperature which preserve its assumed general gamma shape; i.e. fairly uniform density functions with some increased weight given to the lower half of the 1.5°C to 4.5°C range. Not surprisingly, however, the range of marginal damage estimates is extremely sensitive to the specific nonlinearity of the conditional damage function – the $\phi(SLR)$ function characterized in equation (7). If that function were linear in sea level rise, as an extreme example, then expected marginal damages would be a mere 23% higher than the Nordhaus estimate and support only a 7% emissions reduction.

4 UNCERTAINTY ON THE COST SIDE

Consider a global, aggregate production function which relates world GDP at any time t (denoted by GDP (t)) to the employment of labour (denoted by $L(t)$) and the consumption of fossil and/or nonfossil fuel (denoted by $F(t)$ and $E(t)$, respectively).[16] A simple Cobb-Douglas structure was presumed here, so

$$GDP(t) = A(t)L(t)^{sl} F(t)^{sc} E(t)^{sn} \tag{12}$$

where

$$A(t) = A_0(1 + g_a + g_s)^t$$

reflected both the rate of growth of labour productivity (denoted g_a) and the rate of energy-saving technological change (denoted g_s). Labour (population) was assumed to grow over time, as well, at a rate given by g_L, so

$$L(t) = L_0(1 + g_L)^t.$$

Initial conditions in both labour and technology are reflected in L_0 and A_0, respectively; and the parameters sn, sc, and sn in equation (12) reflect the respective shares of GDP devoted to paying for labour, fossil fuel, and nonfossil fuel. The Cobb-Douglas structure assumes that these share remain constant over time, but the inclusion of g_s into the definition of $A(t)$ allows for non-price induced energy conservation.[17]

The two critical prices which drive cost minimizing employment decisions in this model both relate to energy. The price of nonfossil fuel was given by

$$P_n(t) = P_{no}(1 + g_n)^t, \tag{13}$$

where g_n represents a technological bias in the supply of nonfossil fuel; it is positive if technological development over time favours fossil fuel and negative if development favours non-fossil sources. The time trajectory of the price of fossil fuel was taken to be more complicated, depending upon the rate of depletion of the fossil fuel resource stock F and the degree to which that depletion is reflected in the price:

$$P_c(t) = P_{co} + g_1 \frac{\{\mathrm{sum}[F(t)]\}}{\{F - \mathrm{sum}[F(t)]\}} + T, \tag{14}$$

where g_1 is a price-depletion factor, T is an exogenously imposed (carbon) tax, and sum $[F(t)]$ is cumulative fossil fuel consumption from time zero through time t. Initial prices are represented by P_{co} and P_{no} for fossil and non-fossil fuel, respectively.

Cost minimizing conditions required that the ratio of the marginal products of the two types of energy be set equal to the ratio of their respective prices. For the Cobb-Douglas technology given in equation (12), this condition requires simply that

$$E(t) = \frac{snP_c(t)}{scP_n(t)} F(t), \tag{15}$$

so that

$$F(t) = \left\{ A(t)L(t)^{sl} \frac{sc}{P_c(t)} \frac{snP_c(t)}{scP_n(t)}^{sn} \right\}^{(1/sl)} \tag{16}$$

fully characterizes the consumption of fossil fuel over time. Since emissions (denoted $C(t)$) depend upon the consumption of fossil fuel, the link to a emission target could be given by

$$C(t) = C_0(1 + g_c)^t F(t),$$

where C_0 represents an initial ratio of carbon emissions to amount of fossil fuel consumed and g_c represents a rate of change in that ratio over time.

The precise values to be assumed over the next half century or so by many of the parameters specified in this simple model are, of course, unknown. Researchers have, at best, only subjective views of the relative likelihoods for these values; they are, from our current perspective, simply random variables for which we can but suggest subjective probability distributions. The original Nordhaus-Yohe analysis included ten random parameters, but their probabilistic Monte Carlo simulations produced a ranking of the most important sources of uncertainty in our understanding of the likely trajectory of global carbon emissions.[18] Of these original ten variables, the four most important are reflected in the simplified model, constructed here: rate of growth of labour, rate of growth of productivity, technological bias toward or away from non-fossil fuel, and the depletion factor reflected in the price of fossil fuel. Sampling over these four can therefore provide a reasonable presentation of the uncertainty with which we can view the trajectory of future carbon emissions.

Table 2 records three values for each of these parameters.[19] Review of the published literature available in 1983 suggested that subjective relative likelihoods of 25%, 50% and 25% could be associated with the low, middle and high values of each. Simulations based on equations (12) to (16) given these relative likelihood values were undertaken. Table 3 provides summary statistics for the unconstrained scenarios produced by those simulations. It should be noted that the mean trajectory lies slightly above the median. Both track relatively high early in their runs, but lie only slightly above the median trajectory reported by eight modellers participating in EMF-12 by the year 2050. In fact a summary representation of the EMF-12 modellers' unconstrained emissions scenarios from Gaskins and Weyant [5], shows that the 10th and 90th percentile trajectories which emerged from the simulations and which are reported in Table 3 match the boundaries of the EMF-12 runs.

Table 2 *Parameterization of the simplified Nordhaus-Yohe model.*

Parameter	Probability	Value (1990-2000)	Value (2000-2050)
Population growth[a]			
High	0.25	2.0	1.6
Middle	0.50	1.7	1.1
Low	0.25	1.4	0.6
Labour productivity[b]			
High	0.25	3.4	2.3
Middle	0.50	2.3	1.6
Low	0.25	1.2	0.9
Energy price bias[c]			
High	0.25	0.5	0.5
Middle	0.50	0.0	0.0
Low	0.25	–0.5	–0.5
Depletion factor[d]			
High	0.25	612	612
Middle	0.50	342	342
Low	0.25	72	72

Notes:

(a) Recorded as % growth in global population per year.
(b) Recorded in % growth in the GDP/population ratio per year.
(c) Recorded in terms of an annual percentage change in the price of fossil fuel relative to the price of nonfossil fuel.
(d) The g_1 parameter in the supply equation for fossil fuel associated in Nordhaus and Yohe [13] with a fossil fuel resource stock of 11×10^{12} metric tons of coal equivalent.

Table 3 *Unconstrained global emissions of carbon (1990 = 100)[a].*

Year	Median	Mean	10th percentile	90th percentile
1990	100	100	100	100
1995	118	118	111	126
2000	135	136	120	152
2005	154	156	128	183
2010	174	178	136	219
2015	196	201	144	258
2020	205	210	145	275
2025	212	218	145	291
2030	218	225	145	305
2035	225	232	146	319
2040	231	239	146	332
2045	237	245	146	344
2050	242	250	147	354

Note:

Computed, according to the EMF-12 indexing convention that 1990 emissions equal 100, from probabilistic scenario analysis using the Nordhaus-Yohe model [13] as described in Section 4 with the parameters defined in Table 2.

The left-hand side of equation (6c) called for the marginal cost of optimally achieving a specified percentage reduction in cumulative emissions. Appropriate marginal cost schedules for each simulated scenario were therefore required, and they had to incorporate a mechanism by which specified reductions against known emissions totals for each of the unconstrained simulated scenarios were optimally distributed over time. For any scenario, however, this complex allocation problem was nothing more than an intertemporal optimization problem with a limit on total emissions serving as a 'nonrenewable resource' constraint. This is a classic resource problem, of course, whose solution focuses attention upon computing an initial scarcity rent (a shadow price estimate of current marginal cost) for the constrained resource (total emissions, in this case) which then grows over time at the rate of interest.

This notion of a scarcity rent was applied directly to equation (14). For any targeted $x \cdot 100\%$ reduction in cumulative emissions along any simulated future, to be more specific, it was enough to compute some $\lambda_o(x)$ such that inserting

$$T = T(t; x) = \lambda_o(x)e^{rt} \quad (17)$$

into equation (14) would constrain cumulative emissions to $(1\text{-}x) \cdot 100\%$ of the total produced along the unconstrained trajectory. Table 4 provides the summary statistics of the various $\lambda_0(x)$ for the simulations which supported Table 3 – estimates of the current marginal cost, expressed in terms of metric tons of carbon emitted, which achieve dynamic optima through the workings of equations (14) and (17). Notice that the range of uncertainty around these marginal cost estimates is remarkably small [as reflected by the large '*t*-statistics' reported in Column (5)] and that it declines as the targeted percentage reduction in cumulative emissions increases.[20]

The first phenomenon – the almost negligible uncertainty – can be explained with one simple observation. With percentage reduction targets tied to the unconstrained trajectories of individual scenarios, absolute reduction targets are peculiar to that scenario. Even though a 10% reduction along a high growth scenario (e.g.) would involve a significantly larger reduction in absolute emissions than a 10% reduction along a low growth scenario, the larger reduction along the high scenario would be proportionate to the smaller reduction along the low scenario. It should be expected, therefore, that both could be achieved by changes in the relative price of fossil fuel which were very similar.[21]

The second observation – the observed smaller uncertainty for more ambitious reductions – is even easier to understand. It is the immediate result of the decomposition of the delivery price of fossil fuel into its supply price component and its emissions tax, shadow price component. The larger the targeted reduction, the higher the marginal cost of achieving that reduction, the larger the implied tax on fossil fuel, the larger the proportion of the delivery price of fossil fuel attributable to that tax, and the smaller the proportion of the delivery price subject to the uncertainties of the economic modelling of the aggregate economic system.

5 MOMENTUM AND TIMING: CHANGING THE PACE OF DOUBLING

The results presented in Table 4 suggest that uncertainty on the cost side of the marginal efficiency conditions should have little effect on the appropriate abatement response expressed in terms of percentage reductions against scenario specific, unrestricted emissions trajectories. Such a conclusion is, however, misleading because different scenarios imply different rates of emissions growth, different concentrations of greenhouse gases, and thus different contributions to long marginal damages. Recall that the marginal damage of current emissions depends upon the discounted stream of marginal damages which will occur in the future, so that the magnitude of these future damages depends upon atmospheric concentrations yet to be experienced. As a result, early emissions along high emission trajectories must be judged to be relatively more troublesome because their damage must be 'assessed' in terms of the higher costs which will be associated with their corresponding higher concentrations down the line. Conversely, early emissions along a low emission trajectory will be relatively less damaging for the opposite reason.

The implications of this association can be judged quantitatively by making an adjustment to equation (6b) and tracing its effect on the right-hand side of equation (11a). Suppose, to that end, that dE_0 represents emissions along the baseline, median trajectory defined by assuming the median values for each of the driving variables of the emissions model described in Section 4.[22] Regression analysis of any other trajectories (say trajectory z) produced by some other combination of driving variables suggests that each can be represented by

$$dE = \left[dE_0\right]\gamma(z)e^{\rho(z)t}, \tag{18}$$

Equation (6c) can be altered, in light of equation (18), to read

$$g'(E^*) = \mu b\phi'(T^*)\left[\gamma(z)A'(z)\right] \tag{6b'}$$

for emissions trajectory z where the last term, $A'(z)$, is now given by

$$A'(z) = \frac{1}{r-h+\delta-\rho(z)} - \frac{1}{r-h+\delta+a-\rho(z)}. \tag{6c'}$$

It should be expected that $\left[\gamma(z)A'(z)\right]$ would be higher for emissions trajectories above the median baseline reflected in equation (6b) and lower for trajectories below the baseline. Since the mean emissions trajectory lies above the median (see Table 3), it should also be expected that working this complication into the expected value calculations of equations (11a) and (11b) would enlarge, at least slightly, the right-hand side – i.e., the expected marginal damage curve.

Table 5 records the results of adopting the structure of equation (18) and applying equations (6b′) and (6c′) to the right-hand side of the Nordhaus efficiency condition. Table 5 records (in column 1) the characterization of the Nordhaus estimation of the median marginal damage schedule found in Table 1. It also records estimates of

$$\gamma(z), A'(z), \text{ and } \left[\gamma(z)A'(z)\right]$$

for the mean emissions trajectory (column (2)), the 10th percentile emissions trajectory (column (3)), and the 90th percentile emissions trajectory (column (4)).

Table 4 *The marginal cost of emissions reduction against unconstrained trajectories - taxes per ton of carbon emitted.*

% Reduction[a]	(1) Median	(2) Mean	(3) 10th percentile	(4) 90th percentile	(5) t-Statistic
10%	9.92	10.17	8.83	11.45	10.9
20%	22.02	22.48	20.05	24.91	13.1
30%	37.44	38.10	34.72	41.49	15.9
40%	57.89	58.82	54.73	62.91	20.3
50%	86.74	87.99	83.53	92.44	28.0

Note:

(a) Computed as a proportion of unconstrained emissions along the indicated trajectory. The values recorded are denominated in 1989 dollars and represent the efficient scarcity rent defined according to equation (17).

Table 5 *Conditional marginal damage estimates.*

	Marginal damage estimatesa[a]			
T_d/T_o	(1) Median	(2) Mean	(3) 10th percentile	(4) 90th percentile
0.6	5.84	6.17	3.17	11.99
0.7	7.62	8.05	4.85	15.65
0.8	10.41	11.00	6.62	21.37
0.9	16.26	17.18	10.34	33.39
1.0	18.03	19.05	11.47	37.02
1.1	21.34	22.54	13.57	43.87
1.2	25.53	26.97	16.24	52.42
1.3	30.73	32.46	19.54	63.10
1.4	37.72	39.85	23.99	77.45
1.5	47.88	50.58	30.45	98.31
1.6	64.39	68.02	40.95	132.21
1.7	96.65	102.10	61.47	198.45
1.8	190.50	201.25	121.25	391.15
$\gamma(z)$		1.01	1.20	1.10
$A'(z)$		40.01	24.23	78.23
$\gamma(z)\,A'(z)$[b]		40.3	24.2	78.2

Notes:

(a) The values recorded here are denominated in 1989 dollars based upon extrapolation of the Nordhaus [11] and Yohe [23] estimates as outlined in Section 5.

(b) The values of these products were compared with the middle Nordhaus estimate of 38.1 for the discount factor A in equation (6d).

Table 6 *Efficient emissions reductions conditional on expectations about doubling temperature.*

T_d/T_o	*Efficient emissions reduction*[a]			
	(1) *Median*	(2) *Mean*	(3) *10th percentile*	(4) *90th percentile*
0.6	3%	3%	1%	6%
0.7	4%	4%	2%	7%
0.8	5%	5%	4%	10%
0.9	8%	8%	5%	15%
1.0	9%	9%	6%	17%
1.1	10%	11%	7%	20%
1.2	12%	13%	8%	23%
1.3	14%	15%	9%	28%
1.4	17%	19%	12%	33%
1.5	21%	23%	14%	41%
1.6	27%	29%	19%	51%
1.7	38%	42%	27%	64%
1.8	61%	65%	48%	86%

Note:

(a) The values recorded here are expressed as percentage reductions against unconstrained emissions trajectories; see note a of Table 1 for details of the procedure.

The last two columns therefore define the boundaries of what might be considered an 80% confidence interval for the applicable marginal damage schedule contingent upon a range of expectations for the doubling temperature.

Table 6 translates these statistics into efficient emissions abatement targets for the same range of anticipated doubling temperatures. Notice that little change is felt moving from the Nordhaus median baseline, amended to reflect uncertainty in the physical or economic consequences of doubling, to the mean. Dramatic changes do emerge, however, when either the Nordhaus median estimate, or the mean is compared with the 10th and 90th percentile targets. Table 6 can, in fact, be read to suggest that chances are around 1 in 10 that the efficient target would be higher for any anticipated doubling temperature than the values recorded in column (4) and 1 in 10 that the efficient target would be lower for any anticipated doubling temperature than the values recorded in column (3). The mean marginal damage estimate along the 90th percentile trajectory in column (4) is, in fact, roughly \$69 (up from the \$34 estimated produced when nonlinearities and uncertainties were incorporated into the damage side of the calculus in Section 3 and up from the original \$12 Nordhaus point estimate). This estimate of marginal damage supports an efficient emissions reduction target of 29% (up from 15% and 4.5%, respectively); and it would amount to something on the order of 10 to 15 cents per gallon of gasoline. The mean marginal damage estimate along the mean trajectory is almost \$36, supporting a more modest reduction target of only 16%.

6 CONCLUDING REMARKS

Incorporating subjective distributions of the uncertainty with which the future effects of global change phenomena are viewed currently is a difficult process even before nonlinear impacts are added to the calculus. Proper evaluation across a range of possible futures requires, at the very least, some understanding of how a schedule of impacts and potential damage (net of adaptation, but including the cost of adaptation) might be constructed over a range of foreseeable outcomes. These schedules have, for the most part, not yet been constructed. The exaggerated effects and potential damages associated with the lightly weighted, but perhaps highly correlated tails of existing subjective distributions suggest, however, that devoting scarce research resources to that end might pay dividends over the long run.

The present paper supports this contention by exploring three ways in which uncertainty might alter the efficient abatement response to the threat of global warming:

1. the degree to which extrapolating the basic form of an available schedule of economic vulnerability to greenhouse induced sea level rise might effect the expected marginal economic damage of anticipated physical impacts;
2. the degree to which uncertainty over how economic forces might push the global energy system into the future might effect the estimation of the marginal cost of achieving a prescribed reduction in cumulative emissions relative to any specific unconstrained emissions trajectory; and
3. the degree to which the different pace of emissions over future time might effect the discounted stream of these marginal damages estimates.

Correlated subjective distributions of temperature sensitivity and associated sea level rise were employed in Section 3 to reflect both the potential for nonlinear damage and the possibility that uncertainties might cascade to add more weight to the fortunate coincidence of extreme events. Probabilistic scenario analysis was employed in Section 4 to explore uncertainty on the cost side. The same probabilistically weighted scenarios also provided insight into the pace of various possible emissions trajectories in Section 5 – pace which was seen capable of making potentially large contributions to enlarging the economic ramifications of global change.

Feeding correlating distributions of sources and impacts into nonlinear damage schedules increased the baseline marginal damage estimate offered Nordhaus [11] by more than 180% along the mean trajectory, and thereby increased the corresponding efficient reduction in cumulative carbon emissions for the United States from 6% to roughly 16%. Along the 90th percentile trajectory of emissions, marginal damages were increased by another 92% to that the efficient reduction in cumulative carbon emissions would climb close to 30%. Coupled with a complete phase-out of CFC consumption and something around a 1% reduction in carbon emissions produced by carbon sequestering in managed forests, adding uncertainty, nonlinearity and pace to the calculus brings the efficient cumulative reduction in the emission of greenhouse gases through the year 2050 to almost 43%. It must be noted, however, that even this higher response falls well short of the 10% or 20% reductions in emissions relative to 1990 levels which have been discussed in many quarters.[23]

Besides adding some weight to the claim that abatement policies designed to elicit more substantial emissions reductions should be supported, the results reported here suggest something more fundamental for the conduct of research into issues of global change. Analyses of these sorts of issues are typically so involved that careful attention can be paid to only a very limited number of possible futures. Best guess scenarios have typically been selected, especially when time and resources reduce this number to one, but that might be a mistake. Taken qualitatively, the results reported above suggest that focusing on a scenario which describes something around the 75th percentile of potential economic damage might be a better choice – a better reflection of the potential significance of expected damage computed to include the coincidence of 'bad-news' tails.[24] Adding the pace of emissions to the calculation suggests that moving beyond the 75th percentile damage estimate might even be appropriate. At the very least, the construction of an uncertainty multiplier which measures the added expected cost of a wide range of extreme outcomes has been shown to be a productive tact with which to surround either a 'best guess' or a '75th percentile' trajectory with some illustrative measure of the uncertainty with which the future is viewed.

It should be noted, of course, that all of the analysis presented here was built upon the house-of-cards of oversimplification. The original analysis offered by Nordhaus [11] abstracted from the complexity of the natural and social process which drive global change. It also assumed that the composition of the United States economic in the year 2050 will look like the composition of 1981. It ignored investment possibilities, even as it looked at the trade-off between current and future consumption, and it ignored other market failures which might increase or reduce the degree of efficient response to greenhouse warming. The extension presented in Section 3 added to the list of oversimplifications by extrapolating the shape of an aggregate damage function from the shape of the function relating sea level rise to economic vulnerability. It also linearized some complicated structures and assumed risk neutrality in the social objective function. The aggregate model presented in Section 4 meanwhile sacrificed understanding of how various administrative schemes might improve the efficiency of a global abatement policy for the sake of improved understanding of the simulation results.

Allowing more structural flexibility could certainly reduce damages, but adding risk aversion would increase their current welfare cost but not necessarily their discounted values. The net effect of all of this simplification has likely been significant, but the direction in which uncertainty would move the efficient response if more complete representations of what might happen were included in the analysis is unknown. One lesson is clear nonetheless. A strong case can be made that efficient abatement targets are extremely sensitive to systematic inclusion of nonlinear damages and cascading uncertainties even without significant aversion to risk. This sensitivity alone should make us increasingly reluctant to endorse unconstrained alteration of the global environment by sanctioning unconstrained emissions of greenhouse gases.

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ENDNOTES

1 See EPA [2] and EPA [3], for example.

2 See Rosenberg and Crosson [15] for a description of the 'MINK Project' – a research effort which explicit incorporated adaptation; see, as well, NAS [9] for a more thorough review of adaptive possibilities.

3 See Yohe [22, 23] and Shlyakhter and Kammen [17], for example.

4 See NAS [8] for a description of mitigation strategies. See Gaskins and Weyant [5] for a review of the EMF-12 project to which 14 different modellers explored not only the cost of achieving certain carbon emissions targets relative to 1990 levels, but also how sensitive those costs were to changes in how abatement policies were administrated and how the revenue that such policies might collect were distributed.

5 It will not, however, deal with the issue of time horizon raised in Cline [1].

6 See Nordhaus [11] for a complete description of the dynamic optimization procedure employed here. The model described there and employed here is a precursor to the DICE Model developed subsequently by Nordhaus; see Nordhaus [12].

7 Abstracting to a risk neutral objective function does not, of course, eliminate the need for specifying a real discount factor - the pure rate of time preference with which the present value of future consumption is computed. Risk neutrality does, however, imply that the elasticity of marginal utility with respect to per capital consumption is zero. The real rate of return on investment, denoted r, should therefore match the pure rate of time preference, the β parameter in equation (5).

8 Notice that damages associated with the marginal unit of emissions are computed as the discounted sum of a stream of future damages. They depend upon future atmospheric concentrations, as a result, and are therefore sensitive to the trajectory of future emissions.

9 See Yohe [23] for a more complete development of the analysis reported here.

10 See Yohe [21] for the supporting data. The cost of sea level rise along unprotected coastline sums with the cost of protection to quantify the dominant source of potential damage for the United States in the Nordhaus work. Assuming that this dominance persists over the range of possible futures and that the cost of protection rises proportionately with the economic vulnerability of effected locations, advancing equation (7) as a rough approximation of at least the proper form of the marginal damage component of equation (6c) is not totally unwarranted. Its structure is consistent, at the very least, with the notion that increasingly severe changes in climate should move the earth along nonlinear damage functions because they will be associated with increasingly frequent episodes of costly effects and adaptation.

11 Given any specific expectation about the temperature sensitivity of doubling, recent work by Wilson [20], Oerlemans [14] and Shlyakhter and Kammen [16] suggests that an exponential distribution of predicted sea level rise is most appropriate.

12 It has been suggested by some readers of Yohe [23] that truncating the range of possible increases in global mean temperature resulting in equilibrium from an effective doubling of carbon dioxide concentrations (at 1.5 degrees on the low side to 4.5 degrees on the high side) is too restrictive. They argue that a more correct interpretation of the IPCC temperature range would view it as an 80% confidence interval with 10% likelihood remaining that the true change in temperature would lie above or below the designated limits. This work continues to leave exploration of that interpretation to future consideration.

13 The one exception is δ, the atmospheric decay parameter which is small relative to $(r\text{-}h)$ and negatively correlated with b, the airborne fraction parameter. The value assumed by this airborne fraction is generally uncorrelated with the doubling sensitivity of the climate, since it effects a process which occurs before radiative forcing occurs. It is therefore unlikely that either b or A will systematically influence either the $\{T_d/T_o\}$ index or the marginal damages associated with a particular climate change effect, so uncertainty in both will be ignored.

14 This value is computed, by definition, as the middle value reported by Nordhaus ($12.70) times the multiplier 0.46.

15 The marginal cost figures used to support these reduction percentages are drawn, once again, from the original Nordhaus work. His Figure 5, in particular, displays a composite marginal cost curve which is the result of a log-linear regression of emissions reductions against estimated cost run on data recorded in published long run carbon emission scenarios; footnote 10 in Nordhaus [11] for a more complete description of this regression.

16 The analysis conducted here depends critically on the modelling developed to provide probabilistic scenario analysis of future global emissions of carbon dioxide; see Nordhaus and Yohe [13] for details.

17 In the parlance of Manne and Richels [6], non-price induced energy conservation comes under the heading of autonomous energy efficiency improvement.

18 See Table 2.1 in Nordhaus and Yohe [13].

19 These values are taken from Table 2.14 in Nordhaus and Yohe [13]. The parameter g_S takes a fixed value equal to –0.7% change per year to reflect the 30 per cent decline in the world energy-GDP ratio that is emerging as a consensus among modellers participating in the Energy Modeling Forum - 12 investigation into the cost of various carbon emissions reduction strategies; see Weyant and Sit [19] for early discussions; Gaskins and Weyant [5] provides details.

20 The statistics recorded in Table 4 reflect a conversion from a tax denominated in metric tons of coal equivalent computed from the model working through equation (14). An average of 600kg of carbon per metric ton of coal equivalent was assumed (see Nordhaus and Yohe [13]). It should be noted that the mean marginal cost trajectory recorded in column (2) would produce an average tax of approximately $120 per ton of carbon emitted from the year 2000 through the year 2020 along the median economic trajectory. This is an estimate which falls slightly on the high side of the middle range of estimates offered by participants in EMF-12 (see Gaskins and Weyant [5]). The same taxes can be denominated in tons of carbon dioxide emitted by noting that carbon, with an atomic weight of roughly 12, constitutes (12/44) of the molecular weight of carbon dioxide. The resulting transform mean conforms well with the N-Y marginal cost schedule traced by the '+' designation below the regressed marginal cost curve in Figure 5 of Nordhaus [11].

21 Were the reductions expressed in terms of limiting emissions to targets defined as percentages of emissions along a specific future scenario, variation in marginal cost would be much larger across alternative simulations. High growth scenarios would then be faced with disproportionately large absolute emissions reductions which would carry disproportionately high marginal cost; low growth scenarios would, accordingly, face low emissions reductions targets with corresponding low marginal cost. Yohe [24] displays the wide variability for, e.g., a 20% reduction in emissions from 1990 levels.

22 This median baseline trajectory is associated here with the Nordhaus point estimate which equates the marginal cost of the last unit of emission with its corresponding marginal damage.

23 Stabilizing emissions of carbon dioxide at 1990 levels would, for example, be consistent with a 45% reduction in cumulative carbon emissions through 2050 along the median emissions trajectory.

24 Yohe [22] suggests a method by which this sort of information about the distribution of future trajectories can be used to identify useful, 'interesting' scenarios. Note here, for example, that the mean efficient response along the mean marginal damage schedule (a 16% reduction in carbon emissions supported by a marginal cost - scarcity rent of nearly $36) is the scenario-specific efficient

response for a doubling temperature index of between 1.3 and 1.4 (i.e., a doubling temperature expectation of 3.25 to 3.5 degrees centigrade). This expectation lies at about the 75th percentile of the doubling temperature distribution described in Section 3.