Towards a General Comparison of Price Controls and Quantity Controls under Uncertainty
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# Towards a General Comparison of Price Controls and Quantity Controls under Uncertainty

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## 1. INTRODUCTION

In a recent article published in this *Review*, Professor Martin Weitzman [8] developed a cost-benefit model designed to explore the general belief held by most western economists that price controls are a more efficient means of regulation than the corresponding quantity controls. Weitzman effectively concentrated his attention on this preference by analysing only the second best choice between singular, once and for all controls of a production activity. Our present purpose is to extend the Weitzman analysis of the single firm case to include additional sources of uncertainty and informational difficulty that might naturally appear within a regulated heirarchy.[1] In so doing, we suggest a slightly different, but more accurate interpretation of the results that can free us from the assumption that the regulated firm be a profit maximizer.

Our initial generalization appears in Section 2. Unlike the Weitzman model, we do not assume that quantity orders automatically yield the prescribed output with absolute certainty; we postulate, instead, the existence of events that create random discrepancies between such orders and the actual output achieved by the firm. Such distortions in output can quite easily occur despite the best efforts of the firm's manager to meet the order exactly. Since the events we mean to model should influence costs as well as output, the random variables that index them are also exhibited in the argument of the firm's cost schedule. While the importance of the comparison still depends on the relative curvatures of the cost and benefit functions, it is now found to turn on the relative sizes of the variance in output under the two opposing modes of control. We thereby justify a modified interpretation of the basic Weitzman result in terms of the variance in output under prices rather than the variance in marginal cost. It is hoped that this extension will satisfy those who noted in various seminars that the asymmetric influence of uncertainty in the original model biased the results in favour of quotas.

Subsequent sections explore the impacts of adding third order terms to the incumbent approximations and changing the accuracy of the informational heirarchy. The latter are motivated by the informational difficulties that may be encountered when the analysis is applied to a specific situation. The fundamental results of Section 2 are found substantively unaltered in all cases. The possibility that either the centre or the firm is forced to act on the basis of inaccurate information is, for example, seen only to require that the crucial output variations be measured around the incorrect means.

## 2. THE BASIC MODEL

Our variant of the single firm model postulates costs and benefits depending not only upon the quantity of a particular good $q$, but also upon two sets of random variables. Indexing the variables that influence costs and benefits by $(\theta, \xi)$ and $\eta$, respectively, we can write:

$$B = B(q, \eta),$$

and

$$C = C(q, \theta, \xi).^2$$

These variables are envisaged to represent random states of nature that disturb the cost and benefit schedules and which cannot be observed by the central regulating authority before it makes its control decision. They are jointly distributed by $f(\theta, \xi, \eta)$. The centre will attempt to set the production of $q$ near the socially optimal level where benefits minus costs is maximized by issuing either a single valued quantity order or a single valued price order. The singular nature of these potential controls allows us to concentrate our study on their relative merits.

In response to a price order, the peripheral firm is assumed to be capable of observing the actual disturbances in its cost schedule before it selects its profit maximizing output. Production will therefore depend upon the state of nature that actually occurs on the cost side. The same firm is confined to its cost schedule when it faces the alternative quantity control. Weitzman had assumed that it will always be successful in producing exactly the prescribed amount. There may, however, often exist random disturbances that appear after such an order is issued which render it an economic impossibility. We will take note of this possibility by assuming that output can vary even under a quantity control. Indexing the variables that can cause these disturbances with $\xi$, we make a distinction between the quantity ordered by the centre, $q_p$, and the quantity actually produced, $q_a(\xi)$. In so doing, we assume that these output distorting effects are random and that their influence is additive:

$$q_a(\xi) = q_p + \phi(\xi).$$

Observe that since these variables should also be expected to influence costs, they have been lifted directly from the argument of the cost function.

Under these assumptions, the periphery is not interested in the benefit side of the social welfare function. The variable $\eta$ can thus reflect imprecise knowledge of the benefit schedule as well as random shocks to it. The cost related random variables are not quite as versatile. We have thus far been describing a logical extreme of the notion that the periphery operates with better cost information simply because it is closer to the actual production process. The variable $(\theta, \xi)$ can therefore represent neither unobserved random effects on actual costs nor unknown, random errors in measurement. While these possibilities will be discussed briefly in the final section, $(\theta, \xi)$ must now simply reflect day to day shocks to the cost schedule.

The centre will select the optimal quantity order, $\hat{q}_p$, by maximizing expected benefits minus costs with respect to $q$. An amount $\hat{q}_a(\xi) = \hat{q}_p + \phi(\xi)$ would be produced for any $\xi$. The optimal price order, $\tilde{p}$, is similarly selected under the assumption that the centre knows that the periphery is a profit maximizer. The centre therefore knows that the firm will set actual marginal cost equal to any specified price; its quantity reaction to any such price is implicitly defined by

$$p = C_1(h(p, \theta, \xi), \theta, \xi).$$

Maximization of

$$E(B(h(p, \theta, \xi), \eta) - C(h(p, \theta, \xi), \theta, \xi)) \qquad \ldots(1)$$

with respect to $p$ subsequently generates $\tilde{p}$, and $h(\tilde{p}, \theta, \xi)$ is produced for any $(\theta, \xi)$. It is, however, extremely unlikely that either mode of control would yield the ex-post optimal production of $q$. Any comparison of prices and quantities is thereby reduced to determining which control is expected to come closer to the optimum on the average. Following Weitzman's lead, we investigate this query with the comparative advantage of prices over quantities:

$$\Delta = E((B(h(\tilde{p}, \theta, \xi), \eta) - C(h(\tilde{p}, \theta, \xi), \theta, \xi)) - (B(\hat{q}_p + \phi(\xi), \eta) - C(\hat{q}_p + \phi(\xi), \theta, \xi))).$$

This index is simply the expected level of social welfare achieved under price control minus the expected level achieved under the corresponding quota. Prices are preferred when $\Delta$ is positive; quantities when it is negative.

The comparative advantage becomes most tractable when we expand both costs and benefits around a quantity $\hat{q}_0$ defined such that

$$E(B_1(\hat{q}_0, \eta)) = E(C_1(\hat{q}_0, \theta, \xi)).^3$$

We will assume that the variances of the random variables are sufficiently small to justify halting the Taylor approximations with the third terms (see Samuelson [6]); as a result,

$$B(q, \eta) = b(\eta)+(B'\beta(\eta))(q-\hat{q}_0)+\tfrac{1}{2}B_{11}(q-\hat{q}_0)^2, \qquad \text{...(2a)}$$

and

$$C(q, \theta, \xi) = a(\theta, \xi)+(C'+\alpha(\theta, \xi))(q-\hat{q}_0)+\tfrac{1}{2}C_{11}(q-\hat{q}_0)^2. \qquad \text{...(2b)}$$

The impact of incorporating third order terms in the approximations has been investigated; we will report the results in the next section. A geometric interpretation of the present case will, however, allow us to observe that the substantive conclusions we reach here are unaffected. The reader should also notice that we are limiting the stochastic effects to the intercepts of the marginal schedules; their slopes are fixed across states of nature. Both intercepts are, in addition, bisected into means

$$(C' \equiv EC_1(\hat{q}_0, \theta, \xi) \quad \text{and} \quad B' \equiv EB_1(\hat{q}_0, \eta))$$

and disturbances around those means, $(\alpha(\theta\xi) \equiv C_1(\hat{q}_0, \theta\xi)-C'$ and $\beta(\eta) \equiv B_1(\hat{q}_0, \eta)-B')$. We know, therefore, that

$$E\alpha(\theta, \xi) = E\beta(\eta) = 0,$$

and, because of the definition of $\hat{q}_0$, that $B' = C'$.

### 2.1. *The Optimal Orders*

There is an efficiency loss associated with any single valued quantity order, $\bar{q}_p$, that is indicated by the shaded area in Figure 1 for arbitrary values of $(\theta, \xi, \eta)$. That loss can be represented algebraically by the integral

$$L(\bar{q}_p;\ \theta, \xi, \eta) = -\int_{\bar{q}_p+\phi(\xi)}^{q_{\text{opt}}(\theta, \xi, \eta)} (B(q, \eta)-C(q, \theta, \xi))dq.$$

The centre selects its optimal quantity order by minimizing the expected value of these losses (maximizing social welfare), and faces the first order condition that

$$0 = E(\beta(\eta)-\alpha(\theta, \xi)+B'-C'+(B_{11}-C_{11})(\hat{q}_p+\phi(\xi)-\hat{q}_0)).$$

Since $B' = C'$ and $E\alpha = E\beta = 0$, the optimal quantity order is reduced to

$$\hat{q}_p = \hat{q}_0-E\phi(\xi). \qquad \text{...(3)}$$

We observe that the optimal quota is selected so that the expected value of output is $\hat{q}_0$, even in the face of the disturbance $\phi(\xi)$.

Computation of the optimal price order is slightly more involved. Given the quadratic approximation of costs recorded in (2b), the firm's reaction to any price is simply

$$h(p, \theta, \xi) = \hat{q}_0+(p-C'-\alpha(\theta, \xi))/C_{11}.$$

The maximization given by (1) subsequently reduces to

$$\tilde{p} = C',$$

again because $B' = C'$. The optimal price is therefore also given by the intersection of the expected marginal cost and benefit schedules, and the quantity response for arbitrary $(\theta, \xi)$ is simply

$$\tilde{q}(\theta, \xi) = \hat{q}_0-\alpha(\theta, \xi)/C_{11}. \qquad \text{...(4)}$$

We observe that $\hat{q}_0$ is also the expected value of output under price control.

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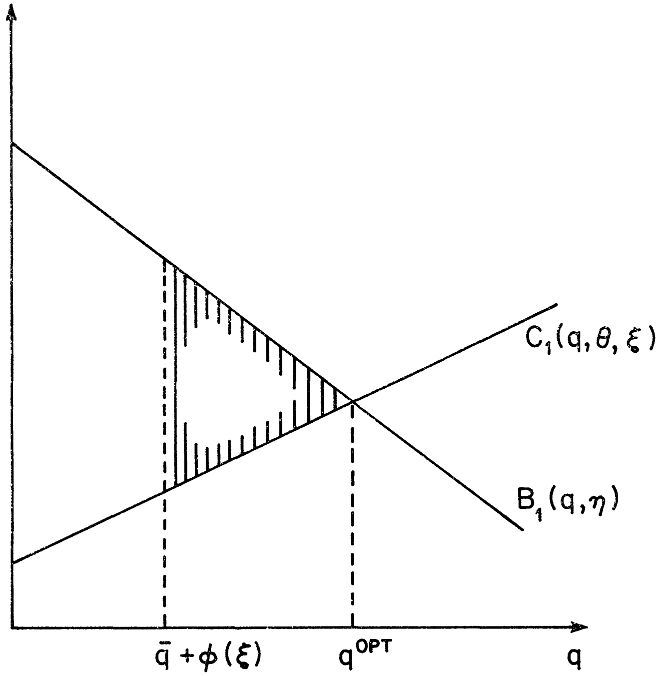


FIGURE 1

### 2.2. *The Comparative Advantage*

Equations (3) and (4) allow us to compute the comparative advantage of prices directly from the definition. Recalling once again that $B' = C'$ and $E\alpha = E\beta = 0$, we are able to show that

$$\Delta = \tfrac{1}{2}(B_{11} + C_{11}) \operatorname{var}\left(\frac{-\alpha}{C_{11}}\right) + \operatorname{cov}\left(\left(\frac{-\alpha}{C_{11}}\right);\ \beta\right)$$
$$-\tfrac{1}{2}(B_{11} - C_{11}) \operatorname{var}(\phi) + \operatorname{cov}(\alpha;\ \phi) - \operatorname{cov}(\beta;\ \phi) \quad \ldots(5)$$

It is also possible to provide an economic interpretation of each of the terms that appear in (5).

Consider, for example, the initial term. The curvature of the benefit function implies that the expected value of benefits under price regulation is less than the level of benefits that would be achieved were the mean output $\hat{q}_0$ produced with certainty. This loss will, of course, increase with both the curvature of the benefit function and the variance in output allowed by prices; it is reflected by $\tfrac{1}{2}B_{11} \operatorname{var}(-\alpha/C_{11})$. The curvature of the cost function similarly implies that the expected value of costs under price control is greater than the level that would be achieved were $\hat{q}_0$ always produced. This loss will also increase with the variance in output, as well as with the curvature of the cost schedule; it equals

$$-\tfrac{1}{2}C_{11} \operatorname{var}(-\alpha/C_{11}).$$

There is, in addition, an efficiency gain under price control that is generated because marginal cost always equals the given price; output must necessarily increase (decrease) just when marginal costs are decreasing (increasing). This is the correct direction, and the expression

$$\operatorname{cov}\left(-\alpha;\ \frac{-\alpha}{C_{11}}\right) = C_{11} \operatorname{var}\left(\frac{-\alpha}{C_{11}}\right)$$

records the incumbent gain. The first term in (5) therefore summarizes the joint influences of the three effects.

Output variation under the quota also causes the mean of benefits to fall below, and the mean of costs to rise above, the levels that would be achieved were $\hat{q}_0$ produced with certainty. Notice, however, that unlike price controls, there is no automatically counterbalancing efficiency gain under quantity control. The expression

$$\tfrac{1}{2}(B_{11}-C_{11}) \text{ var } \phi$$

therefore records these losses and is always negative; it is subtracted in (5) to reflect a positive bias in the comparative advantage of prices.

Our description of the efficiency gain under prices suggests the appropriate interpretation of the covariance terms. The second term, for example, registers the covariance of output variation under price control and the randomly shifting marginal benefit schedule. When this covariance is positive, output tends to increase just as the benefit function reflects greater desire for $q$. Since this is the correct direction for output to move, we note a positive bias in favour of prices. An opposite bias toward quantities is observed when the covariance is negative and output tends to move against the benefit side. Descriptions of the remaining segments of (5) in terms of output variation under quotas are perfectly analogous, and are left to the reader.

Table I reinforces our interpretations by recording the limiting values for $\Delta$ as the curvature parameters approach their extremes.[4] The first two rows are best explained in the context of the particular price reaction function that is implied by our quadratic approximation. Figure 2 illustrates this reaction, and reveals that for a given disturbance in the marginal cost intercept, the resulting output disturbance increases as $C_{11}$ becomes smaller.

TABLE I

*The comparative advantage*

| Limiting factor | Qualification | Reason | $\Delta_1$ |
|---|---|---|---|
| $C_{11}\to 0$ | (none) | Variation under prices is unbounded. | $-\infty$ |
| $C_{11}\to\infty$ | (none) | Variation under prices is negligible. | $+\infty$ |
| $B_{11}\to-\infty$ | var $(-\alpha/C_{11})>$ var $\phi$ | Variation is larger under prices. | $-\infty$ |
| $B_{11}\to-\infty$ | var $(-\alpha/C_{11})<$ var $\phi$ | Variation is larger under quotas. | $+\infty$ |
| $B_{11}\to 0$ | (none) | Covariance effects may dominate. | (amb) |

As marginal costs become horizontal, therefore, output variation under prices becomes arbitrarily large and quantity controls are surely preferred. The opposite conclusion is reached when marginal costs near the verticle; prices are favoured because they induce infinitesimal variation around $\hat{q}_0$.

The final noteworthy extreme is recorded in the third and fourth rows; the benefit schedule is becoming highly curved, and output variation is particularly painful on the benefit side. The sign of $\Delta$ therefore depends crucially upon the sign of

$$(\text{var }(-\alpha/C_{11})-\text{var }\phi).$$

When that expression is positive, the variance in output under price control is greater than the variance under the quota, and quantity control is favoured. The converse, of course, is also true. Notice that the influence of $C_{11}$ is felt even here; as $C_{11}$ increases, the variance in output under prices decreases and it becomes more likely that price control should be preferred.

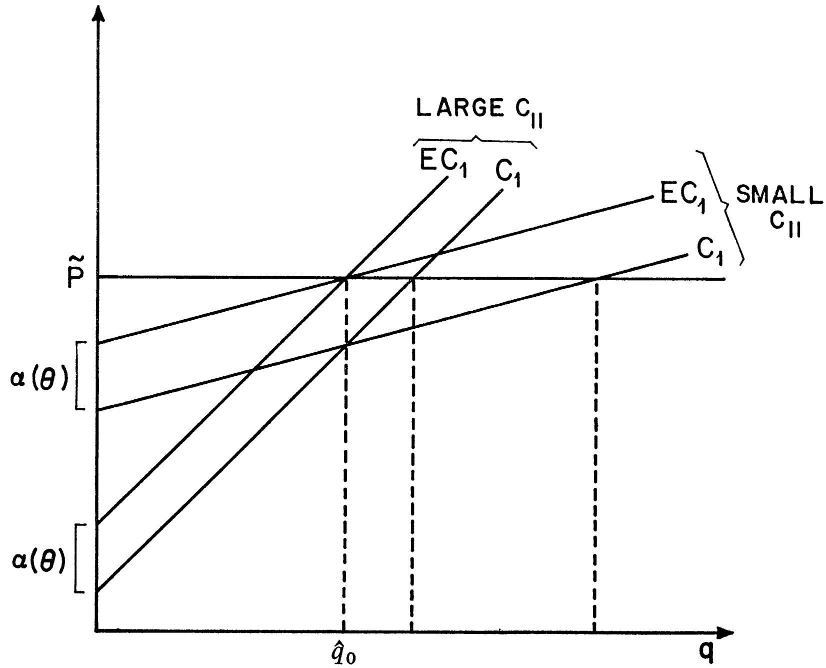


FIGURE 2

*2.3. The Impact of a Capacity Constraint*

We have noted previously that central planners " totally adapt " their quantity orders to the output distortion that they face; i.e. the order is set so that the output mean is precisely equal to the output level that would be required were quotas filled with certainty. It is often argued, however, that planners hedge, in the face of a capacity constraint, against the severe cost penalties of producing above the normal capacity by failing to completely adapt their quantity orders. We can envisage these cost increases emerging from a variety of sources; overtime wages and increased maintenance on overworked and accelerated machinery are but two entries in a long list. In this brief subsection, we examine this conjecture and infer its effect on the prices-quantities comparison.

We can incorporate a capacity constraint into out linear model in the following manner. Define a point $q^{\text{cap}}$ at which marginal costs suddenly become steeper for all states of nature. Figure 3 illustrates our definition for several arbitrary values of $(\theta, \xi)$. If there exists a state of nature such that $\hat{q}_0+\phi(\xi)$ exceeds $q^{\text{cap}}$, then that state is burdened with a penalty of higher costs; this seems to be a reasonable definition of facing a capacity constraint. The shaded area of Figure 3 illustrates the penalty for the lowest marginal cost curve; the penalty is an increase in expected costs equal to

$$E\left\{\int_{\hat{q}_0+\phi(\xi)}^{q^{\text{cap}}} (C_{11}^1+C_{11}^2)(q-\hat{q}_0)dq\right\} > 0.$$

As a result, the optimal quantity order is reduced;

$$\hat{q}_p' = \hat{q}_0 - E\phi - A$$

is ordered where $A \geqq 0$, but is strictly positive when there exists a state of nature such that $(\hat{q}_0+\phi(\xi)-E\phi) > q^{\text{cap}}$. We conclude, as expected, that the centre no longer fully adapts its quotas.

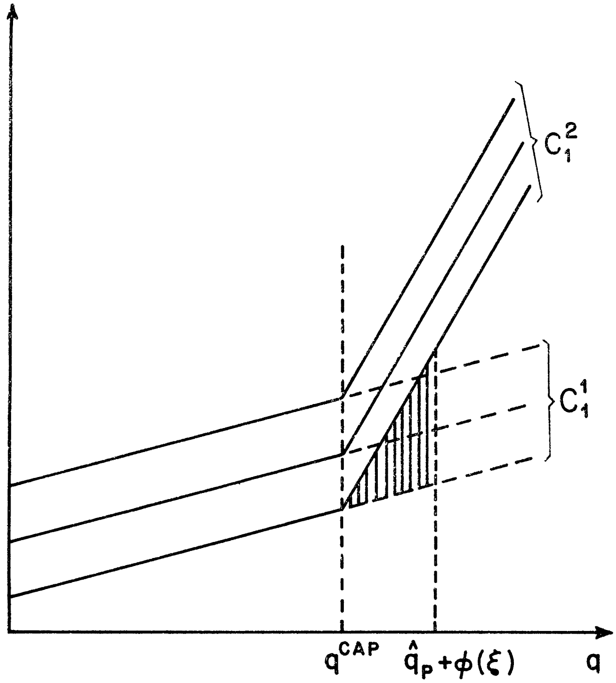


FIGURE 3

The effect on the comparative advantage is also easily deduced. If there exist states of nature in which the optimal price line intersects the steeper segment of the marginal cost curves, then there also exists a positive bias toward prices; output variation under prices would then be less, in these states, than it would have been without the capacity constraint because of the larger slope. This observation strongly suggests that a Soviet planner faced with controlling an over zealous manager should issue price controls and structure an incentive scheme around profits.

## 3. THE IMPACT OF ASYMMETRIC LOSSES

The previous computations have submerged one significant ramification of assuming linear marginal cost and benefit schedules: the loss created by any output disturbance is simply a multiple of $(B_{11}-C_{11})$. As a result, disturbance of equal magnitudes but opposite directions are weighted equally by the social welfare function. To see this point, consider Figure 4A in which $\xi_1$ and $\xi_2$ are defined so that $-\phi(\xi_1) = \phi(\xi_2)$. Areas 1 plus 2 represent the loss associated with $\xi_1$; algebraically,

$$L(\xi_1) = -\tfrac{1}{2}((\phi(\xi_1))^2 \tan \pi + (\phi(\xi_1))^2 \tan \delta)$$
$$= (B_{11}-C_{11})(\phi(\xi_1))^2.$$

Areas 1′ and 2′ similarly reflect the loss associated with $\xi_2$:

$$L(\xi_2) = (B_{11}-C_{11})(\phi(\xi_2))^2 = (B_{11}-C_{11})(\phi(\xi_1))^2.$$

But are large inventories as deleterious as shortages of equal magnitude? If we think not, a strong argument can be offered in support of introducing third order terms into the approximations of costs and benefits. Figure 4B suggests the effect of that change on the

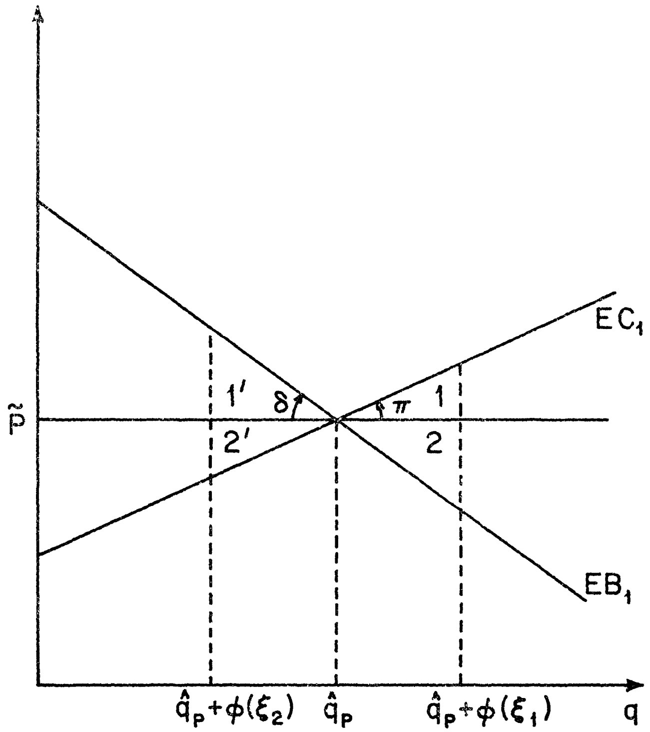


FIGURE 4A

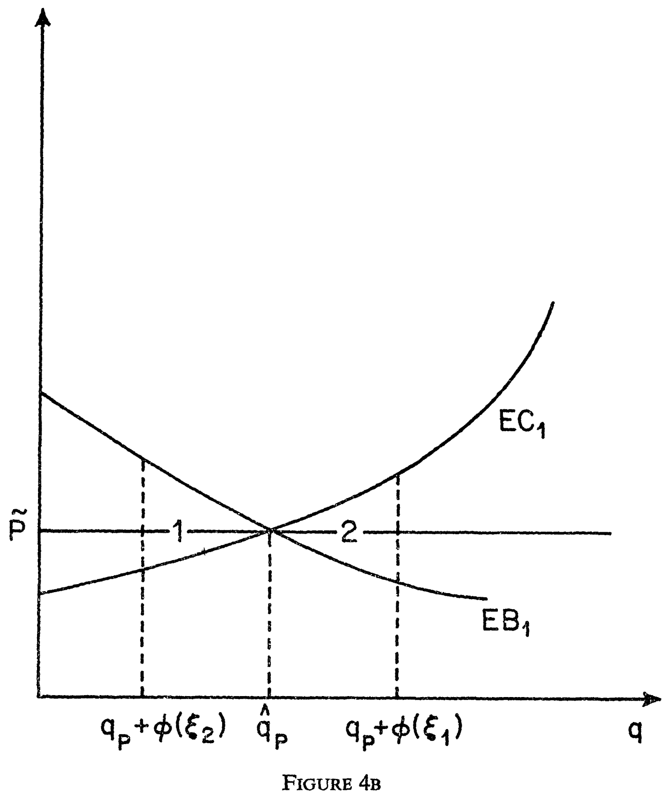


FIGURE 4B

shapes of the marginal schedules and the resulting changes in the loss areas; shortages are shown to be the more serious disturbance.

The mathematics required to handle third order terms is difficult and tedious.[5] The geometric intuition derived from Figures 4 and our emphasis on output variation in the previous section can, however, allow us to deduce their impact without resorting to the algebra. We have observed that the severity of a loss in expected benefits or an increase in expected costs created by output variation depends crucially upon the curvatures of the respective schedules. Third order terms simply reflect the degree with which these curvatures change as output changes. If output variation were skewed toward the more serious side of the mean output where the loss function is more highly curved, for example, then the third order terms would simply register an additional loss in expected welfare. The control that allowed such variation would suffer in the comparative advantage. Variation skewed in the opposite direction would, of course, produce the opposite bias. It should not be difficult, as a practical matter, to infer on a case by case basis when these curvature effects are important and which control they harm more.

## 4. THE IMPACT OF INFORMATIONAL DIFFICULTIES

We have already noted that the basic model is an informational extreme. We now conclude our discussion by exploring briefly two alternative formulations. The first is even more extreme. Suppose that the centre must compute its optimal controls with an inaccurate subjective distribution of the random variables that influence social welfare. An unperceived bias in measurement, an insufficiently fine measurement grid, or simply imprecise reporting of data could certainly produce such inaccuracy. Both control choices would then be incorrectly specified, in most cases, and incur a dead weight loss. Just as they were centred around $\hat{q}_0$ in Section 2, however, response by the firm to these orders would now be centred around the incorrect quota. As a result, these losses would cancel when they are inserted into the comparative advantage. Only output variation would remain to differentiate the control options, but it must be measured around the incorrect mean. The entire analysis of variance and curvature that has been developed can thus be applied directly if we change the expansion points of the approximations.

A second possibility involves inaccurate information at both the centre and the peripheral firm. Suppose that the firm is also forced to make its output decisions before the true values of $(\theta, \xi)$ are known; it would maximize expected profits. Were its subjective distribution for the random variables different from the centre's, an informational asymmetry would be produced. Such an asymmetry is not unreasonable, since the centre and the firm view the world from different perspectives. Since output decisions are made for both controls *ex ante*, we can also assume that $\phi(\xi)$ operates on actual output under both. Dead weight losses are thus the only differences, and should be the subject of the centre's concern. It is intuitively pleasing that the decision maker who makes the smaller error in issuing a quantity order should be allowed to do so. If the error made by the centre in issuing a quota is smaller than the error made by the firm in responding to a price order, the centre should issue the quota. Otherwise, the centre should issue a price control and allow the periphery to choose the intended output, even though the price is surely mis-specified.

In the present extension, the cost variables can finally be thought to represent imprecise knowledge and improper measurement of actual costs; neither decision maker is required to view them and the subjective densities need not be accurate. There are, of course, a multitude of examples that are intermediate to the extremes we have presented: the firm can react to some, but not all of the cost variables as it decides how much to produce under the price control. In such cases, it should seem reasonable that we need compare only the variations in output induced by these restricted reactions and the dead weight losses caused by not being able to react to all of the variables.

## 5. CONCLUSIONS

We have seen that the fundamental Weitzman conclusion survives extensive generalization completely intact: a blanket subscription to price controls in lieu of the alternative quantity controls is economically unsubstantiated. Even when quantity orders produce variation in output, the comparison turns primarily upon the relative magnitudes of such variation. The importance of the comparison, meanwhile, depends crucially upon the curvatures of the cost and benefit components of social welfare. In addition, either mode of control would operate in the context of randomly shifting marginal cost and benefit schedules, and the correctness of the directions of output variation must also be considered. The right direction is, of course, defined by the shifts in the marginal functions, themselves. Each policy choice between price and quantity controls must therefore be made individually on the basis of the expected patterns of output variation associated with the two alternatives.

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NOTES

1. Analysis of the multifirm and multigood cases are topics of sufficient depth to lie outside the scope of a single article. The interested reader is referred to [10], [11] and [12] for applications of the present model to these circumstances.

2. We presume for all $(\theta, \xi, \eta)$ that $B_1(q, \eta)$ and $C_1(q, \theta, \xi)$ are both positive, $B_{11}(q, \eta) > 0$, $C_{11}(q, \theta, \xi) > 0$, $B_1(0, \eta) > C_1(0, \theta, \xi)$, and that there exists a positive real number $N$ such that $q > N$ implies that

$$B_1(q, \eta) < C_1(q, \theta, \xi).$$

The assumptions simply assure a positive optimal output for all states of nature.

3. The assumed shapes of the benefit and cost schedules guarantee that $\hat{q}_0$ exists.

4. The Weitzman results differ when $C_{11} \to \infty$ and when $B_{11} \to -\infty$. The reason, in both cases, is that the quantity orders are produced with certainty in the Weitzman model. Both modes therefore produce $\hat{q}_0$ in the first case, and the comparative advantage is zero. To see the second, simply observe that $\phi(\xi) \equiv 0$, so that var $(-\alpha/C_{11})$ is always larger.

5. A complete derivation of the results reported here can be found in Chapter II of [9].

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