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SINGLE-VALUED CONTROL OF AN INTERMEDIATE GOOD UNDER UNCERTAINTY: A COMPARISON OF PRICES AND QUANTITIES*

BY GARY W. YOHE[1]

The relative merits of single-valued price and quantity controls in an uncertain economic environment have attracted considerable analytic attention in the recent literature.[2] Aside from the few cases in which institutional constraints preclude one type of control or the other, these studies have undermined the conventional western wisdom that prices are always the better regulatory choice. They have exhibited situations in which, *ceteris paribus*, the *ex post* distribution of output created under quantity control is socially preferable to the corresponding distribution created by the certainty equivalent price control. Unilateral imposition of price controls is thus an inferior strategy.

Weitzman [7] prepared the sentinel article of this comparison in 1973. He was able to concentrate on the relative operational merits of prices and quantities by restricting his attention to a central regulatory body that is confined to the issuance of a single, once and for all control order to a profit maximizing producer of a single output. The economist who casually applies the conclusions that can be drawn from general comparisons like Weitzman's to a more structured regulation problem can, however, easily overlook significant effects that may be exerted on his comparison by that very structure. Short of building a full-fledged general equilibrium model, these effects can be captured by incorporating such structure explicitly into an extension of the Weitzman model. The present study is such a construction; we will compare once and for all price and quantity control of an intermediate good that derives its value entirely from the final good it is used to produced. Careful attention will be paid to the impact of the elasticity of substitution in the production of the final good and to the profitability of producing that good in the face of either type of regulation. While it is a tribute to the versatility of the Weitzman framework that such structure can be successfully incorporated, it will become apparent that his basic model falls well short of handling the intricacies of our problem.

Even without discussing the reasons for control, the reader should also agree that this comparison can be of considerable practical importance; sectors of a

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[2] See, for example, Poole [4], Spence and Roberts [6], Weitzman [7], and Yohe [9].

planned economy or of any vertically integrated production unit are but two entries in a long list of potential applications. Even casual observation of these situations will reveal a past preference for simplicity in the chosen control. When General Motors and Ford experimented with alternative schemes in the early 1960's for example, they chose to compare only singlevalued internal transfer prices with the existing quota system (see Whinston [**8**]). Our comparison of the extreme alternatives is thus entirely germane to their choice. Our results should, in addition, be of significance to those regulatory agencies who admit the possibility that a more complex mixed policy may be worth the administrative effort. By computing which pure control is better, the agency should ascertain which is the appropriate basis upon which to build the mix; that is, such information should weigh heavily upon whether one chooses to specify prices for a variety of quantity intervals, or set quotas for some given price intervals.

Our study, then divides itself naturally into two parts. The first, housed in Sections 1 through 5, begins by setting the stage and ends by specifying the optimal price and quantity orders. In between, considerable effort is spent investigating precisely how each firm will react to an arbitrary order of either type. It is these reactions that allow us to compute the best orders. A second concern will be the profitability that these controls allow. The existence of any profit-motivated pressures on the producer of the final good to avoid either control by integrating the intermediate good into his own production process must be carefully noted. Section 6 begins the second part in which the optimal controls are actually compared. The impact of substitution on the comparison is of great interest here, as the distributions of output associated with the two alternatives are evaluated in terms of their imputed expected benefits and expected costs.

1. THE MODEL

We will be calling the final good from which benefits are derived X and the intermediate good that is to be controlled Q. The production of X is summarized by $X=F(Q, K)$, where K is a second input (or an aggregate of several inputs that are employed in fixed proportions) available at a constant per unit price r. For the most part, we will be assuming that inventories of Q are maintained at a fixed level; any fluctuation in the output of Q is thus registered to some degree in the output of X. The impact of relaxing this constraint is, however, easily deduced toward the end of our analysis.

Benefits are presumed to depend on a vector of random variables, η, in addition to X. These variables are meant to reflect imprecise knowledge of the benefit schedule for X as well as desultory shocks to the schedule, itself. Thus,

$$B = B(X, \eta).$$

The production of Q at the Q-firm is meanwhile summarized by a cost function that depends not only upon Q, but also upon a second vector of random variables,

θ. The two vectors are presumed jointly distributed by $f(\theta, \eta)$. In θ, we represent both day to day shocks in the production of Q and the more precise knowledge of the cost schedule that is available to the Q-firm, but not the center. Thus,

$$C = C(Q, \theta).$$

The center must select its order before the actual values of θ and η are known, and therefore does an expected value computation in making its choice. The Q-firm, meanwhile, maximizes profits in response to any price order by reading the true value of θ, and setting actual marginal costs equal to that price. A corresponding quantity order is presumed, for the moment, to be met precisely regardless of θ. Uncertainty about the true cost schedule at both the center and the Q-firm is therefore not captured by θ: such uncertainty, however, exerts a neutral effect on our comparison of prices and quantities and can be ignored (see Yohe [**9**, (Chapter 2)]).

A few technical assumptions can guarantee that the optimal production of Q for any (θ, η) is positive. The benefit function has the usual shape for all (X, η): $B_1(X, \eta) > 0$ and $B_{11}(X, \eta) \leq 0$. The cost function is equally standard, with $C_1(Q, \theta) > 0$ and $C_{11}(Q, \theta) \geq 0$ for all (Q, θ). If we then presume that for every pair (θ, η), $B(0, \eta) > C(0, \theta)$ and $B_1(0, \eta) > C_1(0, \theta)$, our guarantee is completed.

Finally, the reader should have noticed that the behavior of the producer of X (the X-firm) is thus far undefined. Two possibilities are explored in Sections 3 and 4 because it is not clear, at this point, whether that behavior will have an influence on the comparison. Before turning to that question, however, we will show that the elasticity of substitution in the production of X between K and Q does indeed have an impact.

2. THE EXTREME CASES

The simplest way to demonstrate that impact is to present the two possible extremes in juxtaposition:

$$X = \gamma K + (1 - \gamma)Q: \tag{1}$$

$$X = \min\{\gamma K; (1 - \gamma)Q\}. \tag{2}$$

We will assume that the X-firm is publicly spirited in the sense that the K-response to a delivery of Q is determined by maximizing expected social benefits minus private costs. A per unit charge equal to its expected marginal value product is levied for Q in both cases.[3] While these qualifications may now appear to be quite restrictive, subsequent analysis will verify the general validity of the conclusions we will draw. We will also presume, for the remainder of this section,

[3] All that is really required in the second case is a strictly positive price to guarantee that the X-firm operates on the corner of its chosen L-shaped isoquant.

that θ and η are independent.

Observe, in case (1), that the X-firm will respond to a delivery of Q by solving the first order condition

$$E\{B_1[(\gamma\hat{K}(Q) + (1 - \gamma)Q), \eta]\} = r$$

for $\hat{K}(Q)$. The center can view $\hat{K}(Q)$ as given and determine the optimal quantity order, $\hat{Q}$, by solving

$$\max_Q E\{B[(\gamma\hat{K}(Q) + (1 - \gamma)Q), \eta] - r\hat{K}(Q) - C(Q, \theta)\}.$$

The optimal price order, $\tilde{p}$, is similarly computed by noting that the reaction of the Q-firm to any price is defined by

$$p = C_1(Q(p, \theta), \theta).$$

The center must therefore solve

$$\max_p E\{B[(\gamma\hat{K}[Q(p, \theta)] + (1 - \gamma)Q[p, \theta]), \eta] - rK[Q(p, \theta)] - C[Q(p, \theta), \theta]\},$$

since $Q(p, \theta)$ is now delivered to the X-firm.

The amount of X produced under optimal quantity control is thus characterized by

$$E\{B_1[\gamma\hat{K}(\hat{Q}) + (1 - \gamma)\hat{Q}), \eta]\} = r,$$

while the corresponding amount produced under $\tilde{p}$ for an arbitrary θ is characterized by[4]

$$E\{B_1[(\gamma K[Q(\tilde{p}, \theta)] + (1 - \gamma)Q[\tilde{p}, \theta]), \eta]\} = r.$$

But since the benefit function is arbitrary, we may conclude that

$$\gamma\hat{K}(\hat{Q}) + (1 - \gamma)\hat{Q} = \gamma K[Q(\tilde{p}, \theta)] + (1 - \gamma)Q(\tilde{p}, \theta)$$

for all θ; as the states of nature on the cost side change, the amount of Q delivered to the X-firm changes, but K is adjusted so that the output of X remains constant.[5] The output of X is therefore equal to $[\gamma\hat{K}(\hat{Q}) + (1 - \gamma)\hat{Q}]$ regardless of the type of control imposed on the intermediate good. If we now compute the difference between the expected levels of benefits minus costs achieved by $\tilde{p}$ and $\hat{Q}$ (the comparative advantage of prices), we need only consider the cost effects of variation in the production of Q under $\tilde{p}$. To be sure, expected costs exceed the level that is achieved when $\hat{Q}$ is produced with certainty because deliveries vary. It will be shown, however, that the concurrent efficiency gain of always having $\tilde{p}$

[4] In this formulation, the X-firm takes $Q(\tilde{p}, \theta)$ as given and computes the expected value; the expected value operator therefore does not pass into the Q function.

[5] So that we are on the interior of the isocost line, the ratio of input prices must reflect the slope of the linear isoquants of the first case; they must equal $(1-\gamma)/\gamma$.

equal to marginal cost, and thus always having output move correctly with respect to costs, dominates that loss. Price controls are preferred unambiguously in case (1).

The second case specifies that absolutely no substitution be allowed between K and Q in the production of X. Since we have established a price for Q that guarantees that X is produced on the corner of the right angle isoquants, we can observe immediately that $X=(1-\gamma)Q$. The benefit function is thus easily expressed as a function of Q, and it becomes a simple matter to compute either optimal control. The output of good X, however, now varies as the production and delivery of Q varies under prices. The comparative advantage of prices must now capture a loss in expected benefits under prices in addition to the positive cost contribution; its sign is very much in doubt.

These two observations have uncovered a fundamental difference in the comparison of prices and quantities that is created by shifting the elasticity of substitution from one extreme to the other. The remainder of this study is devoted to putting this difference into perspective by considering the intermediate cases. Section 3 introduces these middle examples in the context of our publicly spirited X-firm. Output responses and their imputed expected profits for producing X are explored under a variety of pricing schemes for Q. In Section 4, the producer of X is allowed to maximize expected profits, but the output responses emerge unchanged. As we subsequently turn to the comparative advantage of prices in Section 6, we do so with the knowledge that the underlying quantity responses are sufficiently general.

3. A PUBLICLY SPIRITED PRODUCER OF X

The cases of intermediate substitutability are all represented by CES production functions of the form

$$X = [\gamma K^{\rho} + (1-\gamma)Q^{\rho}]^{1/\rho} \equiv F(K, Q).$$

The elasticity of substitution is then simply

$$\sigma = \frac{1}{(1-\rho)}$$

as ρ ranges from $-\infty$ to 1. The triple $(\hat{Q}_0, \hat{K}_0, \hat{X}_0)$ that maximizes expected social welfare can be determined by solving

$$\max_{Q;K} E\{B[(\gamma K^{\rho} + (1-\gamma)Q^{\rho})^{1/\rho}, \eta] - rK - C(Q, \theta)\}; \tag{1}$$

of course,

$$\hat{X}_0 \equiv [\gamma \hat{K}_0^{\rho} + (1-\gamma)\hat{Q}_0^{\rho}]^{1/\rho}. \tag{2}$$

The reader should note that (1) is a precise statement of the center's objective, since it must act before the true values of θ and η are known. The first order conditions are most easily recorded as follows:

(3a) $$E\{B_1 \cdot F_K\} = r$$

(3b) $$E\{B_1 \cdot F_Q\} = E\{C_1(\hat{Q}_0, \theta)\}.$$

The assumed shapes of the benefit and cost schedules guarantee both the existence of $(\hat{K}_0, \hat{Q}_0)$ and the second order conditions for a maximum. The subsequent analysis will be tractible only if we now make the following second order Taylor series approximations:

(4a) $$B(X, \eta) = b(\eta) + (B' + \beta(\eta))(X - \hat{X}_0) + 1/2B_{11}(X - \hat{X}_0)^2;$$

(4b) $$C(Q, \theta) = a(\theta) + (C' + \alpha(\theta))(Q - \hat{Q}_0) + 1/2C_{11}(Q - \hat{Q}_0)^2.$$

These approximations are mathematically defensible if $f(\theta, \eta)$ is compact and the variances of θ and η are small (see Samuelson [5]). Their cost in terms of omitted economic content has also been explored in a simpler context and found negligible.[6] Observe that the first order coefficients in both schedules have been divided into means and disturbances around those means; for example, $B' = E[B_1(\hat{X}_0, \eta)]$ and $\beta(\eta) = [B_1(\hat{X}_0, \eta) - B']$. As a result, $E\beta(\eta) = E\alpha(\theta) = 0$.

Equations (3) should be expressed in terms of our new approximations:

(3a′) $$E\left\{[B' + \beta(\eta)]\left[\frac{(\gamma\hat{K}_0^\rho + (1-\gamma)\hat{Q}_0^\rho)^{1/\rho}}{\hat{K}_0}\right]^{1-\rho}\right\} = r$$

(3b′) $$E\left\{[B' + \beta(\eta)]\left[\frac{(\gamma\hat{K}_0^\rho + (1-\gamma)\hat{Q}_0^\rho)^{1/\rho}}{\hat{Q}_0}\right]^{1-\rho}\right\} = C'.$$

These revised equations can now be combined to reveal that

(5) $$(\hat{K}_0/\hat{Q}_0)^{1-\rho} = [\gamma C'/(1-\gamma)r] \equiv Z.$$

Subsequent manipulation of (5) allows a convenient condensation of notation:

$$\hat{K}_0 = Z^\sigma\hat{Q}_0, \quad \text{and}$$

$$\hat{X}_0 = \hat{Q}_0[\gamma Z^{\sigma\rho} + (1-\gamma)]^{1/\rho} \equiv \hat{Q}_0 A(\rho).$$

The first order conditions recorded in (3′) therefore require that

(6a) $$B'[A(\rho)/Z^\sigma]^{1-\rho} = r$$

(6b) $$B'[A(\rho)]^{1-\rho} = C'.$$

In the context of the above specifications, we now turn to investigate (i) the reactions of the Q-firm and the X-firm to controls of both types, (ii) the computation of the two optimal control alternatives given those reactions, and (iii) the profitability of producing X in the face of those optimal controls. The

[6] Quadratic functions such as those in (4) have constant curvature throughout their domain. Incorporating third order terms allows that curvature to change, but since, in our study, the outputs of both goods will have the same mean under both types of control, the effect is neutral (see Chapter 2 in Yohe [9]).

X-firm is assumed to be publicly-spirited throughout and, unless otherwise specified, pays a price equal to the expected marginal value product of Q for each unit of Q it receives.

3.1. *Output responses to control orders.* Given any price order and the quadratic cost function listed in (4b), the Q-firm maximizes profits for an arbitrary θ by producing

$$Q(p, \theta) = \hat{Q}_0 + \left\{\frac{p - \alpha(\theta) - C'}{C_{11}}\right\} \equiv \hat{Q}_0 + h(p, \theta).$$

The K response of the X-firm to a delivery of $Q(p, \theta)$, designated $K(h)$, is characterized implicitly by the FOC for maximizing expected benefits minus private costs with respect to K:

$$(7) \qquad E\left\{[B' + \beta(\eta) + B_{11}h(p, \theta)A(\rho)] \left[\frac{(\gamma[K(h)]^\rho + (1-\gamma)[Q(p, \theta)]^\rho)^{1/\rho}}{K(h)}\right]^{1-\rho}\right\} = r.$$

Deliveries under quantity control of the Q-firm are constant, so that the corresponding response to a quota $\bar{Q}$, designated $K(\bar{Q})$, is similarly characterized:

$$(8) \qquad E\left\{[B' + \beta(\eta) + B_{11}(\bar{Q} - \hat{Q}_0)A(\rho)] \left[\frac{(\gamma[K(\bar{Q})]^\rho + (1-\gamma)\bar{Q}^\rho)^{1/\rho}}{K(\bar{Q})}\right]^{1-\rho}\right\} = r.$$

The center must use its knowledge of these reactions to compute the optimal control specifications.

3.2. *The optimal controls.* The best quantity order is the easier to compute. Equation (3a′) guarantees that $\hat{K}_0$ will be selected by the X-firm if $\hat{Q}_0$ is delivered. The desired output, $\hat{X}_0$, would thus be forthcoming, so that $\hat{Q}_0$ is the optimal quota.

The optimal price, $\tilde{p}$, is a bit more trouble, but we will record the solution and appeal to the shapes of the underlying schedules to assert uniqueness. The center is interested in solving

$$(9) \qquad \max_p E\{B[(\gamma[K(h)]^\rho + (1-\gamma)[Q(p, \theta)]^\rho)^{1/\rho}, \eta] - rK(h) - C(Q[p, \theta], \theta)\}.$$

In light of (7), the first order condition for (9) reduces to

$$(10) \qquad E\left\{[B' + \beta(\eta) + B_{11}h(\tilde{p}, \theta)A(\rho)] \left[\frac{(\gamma[K(h)]^\rho + (1-\gamma)[Q(\tilde{p}, \theta)]^\rho)^{1/\rho}}{Q(\tilde{p}, \theta)}\right]^{1-\rho}\right\} = \tilde{p}$$

The left hand side of (10) is the expected marginal value product of Q; even without solving for $\tilde{p}$ we see that the center is optimally buying and selling Q at the same price. Furthermore, the joint assertions that

$$K(h) \equiv Z^{\sigma}Q(p, \theta) = \hat{K}_0 + Z^{\sigma}h(p, \theta), \quad \text{and}$$
$$\tilde{p} \equiv C'$$

guarantee that (10) reduces to (6b) and (7) reduces to (6a). Equality is therefore guaranteed by construction, and the assertions state the optimal price. The resulting outputs of Q and X, and employment of K, are then

$$\tilde{Q}(\theta) \equiv Q(\tilde{p}, \theta) = \hat{Q}_0 - [\alpha(\theta)/C_{11}],$$
$$\tilde{X}(\theta) \equiv \tilde{Q}(\theta)A(\rho) \equiv \hat{X}_0 - A(\rho)[\alpha(\theta)/C_{11})], \quad \text{and}$$
$$\tilde{K}(\theta) \equiv Z^{\sigma}\tilde{Q}(\theta) = \hat{K}_0 - Z^{\sigma}[\alpha(\theta)/C_{11}].$$

3.3. *The profitability of producing X under optimal control of Q.* Since all *CES* production functions obey Euler's theorem, we can observe immediately that

$$(11) \qquad \hat{Q}_0A(\rho) = \left[\frac{\hat{Q}_0A(\rho)}{\hat{Q}_0Z^{\sigma}}\right]^{1-\rho}\hat{Q}_0Z^{\sigma} + \left[\frac{\hat{Q}_0A(\rho)}{\hat{Q}_0}\right]^{1-\rho}\hat{Q}_0.$$

If the selling price for X accurately reflects marginal benefits, multiplying (11) by $[B'+\beta(\eta)]$ and taking expected values produces two expressions for the expected revenue generated by the sale of $\hat{X}_0$. The right hand side emerges from that process, with the aid of equations (3′), as the sum of total expenditures on $\hat{K}_0(rZ^{\sigma}\hat{Q}_0)$ and total expenditures on $\hat{Q}_0(C'\hat{Q}_0)$. Expected profits are therefore zero, and there exist no pressures to avoid the quantity control on Q by integrating its production into the production process for X.

The profitability of producing X under control of Q by $\tilde{p}$ is, however, another story. Euler's theorem can be employed a second time to reveal that expected revenues now equal

$$(12) \qquad B'[A(\rho)/Z^{\rho}]^{1-\rho}\hat{K}_0 + B'[A(\rho)]^{1-\rho}\hat{Q}_0$$
$$+ [A(\rho)/Z^{\sigma}]^{1-\rho}\operatorname{Cov}\{[\beta(\eta) + B_{11}A(\rho)(-\alpha(\theta)/C_{11})]; Z\sigma\tilde{Q}(\theta)\}$$
$$+ [A(\rho)]^{1-\rho}\operatorname{Cov}\{[\beta(\eta) + B_{11}A(\rho)(-\alpha(\theta)/C_{11})]; \tilde{Q}(\theta)\}.$$

The first term of (12) represents the expected total expenditure for K; the second term, the expected total cost of Q. These sum to the total expected cost of running the X-firm. Expected profits are therefore crucially dependent upon the final two expressions in (12); they combine to become the covariance of shifts in the marginal benefit schedule and shifts in the production of X. Output increases, on the average, as the selling price increases only if that covariance is positive. Expected revenues would then exceed expected costs and the firm would average a positive profit. The opposite occurs, of course, when the covariance is negative; there would then exist pressure on the X-firm to avoid this loss situation by manufacturing its own Q. The astute reader will note that the economic feasibility of such vertical integration has just been demonstrated in the preceding paragraph.

Careful examination of the crucial covariances reveals a permanently negative subterm:

$$\mathrm{Cov}\,[B_{11}Z(-\alpha(\theta)/C_{11})];\, Z(-\alpha(\theta)/C_{11})] = B_{11}\,\mathrm{Var}\,[Z(-\alpha(\theta)/C_{11})] < 0\,.$$

This term represents the loss caused by the correlation between changes in output and their induced changes in marginal benefits. A significant positive correlation between these output changes and uninduced changes in marginal benefits would therefore be required to create non-negative expected profits. The mere independence of θ and η could easily create one of the many possible circumstances in which a price control could be preferred, while the expected profits of the X-firm are negative. The profitability of producing X can therefore be a serious problem.

The crucial assumptions of the preceding analysis with respect to profits have been that the X-firm is required to use all of the Q that is delivered, and is charged the expected marginal value product for that Q. The producer of X is therefore able to compute his K response to a delivery of Q independent of its price; he simply takes the delivery as given and maximizes the expected value of his objective function with respect to K. Quite independent of that maximand, then, this K response is invariant across the various pricing policies that may be imposed on Q. As we now turn to engineer changes in that policy in an effort to improve profitability, we need not rework our previous analysis of the X-firm's behavior.

Perhaps the simplest procedure would be to deliver the intermediate good to the X-firm gratis. The above argument implies that the optimal quantity order remains $\hat{Q}_0$, the optimal price order C', and the X response to that order $[\hat{X}_0 - A(\rho)(\alpha/C_{11})]$. There is, however, an average transfer of revenue from the center to the X-firm in the amount $C'\hat{Q}_0$ associated with those deliveries under either type of control. It should be expected that this transfer would render the X-firm profitable, even given the covariance difficulties listed above, thereby eliminating the pressure to integrate the production process.

A policy with less severe distributional effects can, however, be devised. Suppose that the center were willing to postpone payment for Q until after the corresponding X had been produced and the actual marginal value of Q becomes known. The center could then charge the X-firm a per unit fee for Q precisely equal to its actual marginal value product in producing X. All of the optimal orders and output responses would remain the same, but the expected revenues of the X-firm would differ from expected costs by only

$$\mathrm{Cov}\,\{\beta(\eta) + B_{11}[Z^{\sigma}(-\alpha/C_{11})];\, [A(\rho)/Z^{\sigma}]^{1-\rho}(-\alpha/C_{11})\}\,.$$

The troublesome covariance term has been essentially "cut in half," now reflecting only the charging of a nonstochastic price for K.

Reviewing this second pricing variation makes it clear that the difficulty in the profitability of the X-firm under price control of Q lies in the fact that a quantity times an expected marginal product does not necessarily equal the expected value of that quantity times the actual marginal product. Were the center to charge the actual marginal value product of K for each unit of K used in producing X, the production of X would net an expected economic profit of zero under prices as well as quantities. If, in addition, the center were to require that the expected value of the actual marginal value product of K be precisely equal to r, the first order condition of the X-firm with respect to K would remain precisely the same, and the K response to deliveries of Q would also be preserved. This second condition guarantees that the center, in buying K at r and selling it at its actual marginal value product in the production of X, would break even on the average.

The preceding paragraphs have recorded a pricing policy that both solves the profitability problem under price control and leaves the behavioral analysis of the first two subsections unaltered. It will, however, greatly facilitate the exposition of further complications in the model if we are consistent and continue to charge the X-firm per unit charges equal to the expected marginal value product for Q and r for K. While we maintain that presumption throughout the remainder of this study, the reader should always be aware that the subsequent analysis is equally valid in the context of pricing schemes that guarantee the profitability of producing X without integration.

4. A PROFIT MAXIMIZING PRODUCER OF X

We can easily extend our analysis to model a profit maximizing producer of X who is equipped with only a distribution of the price at which he believes he can sell his product. In particular, we assume that he knows that the price varies with the vector η, and that he thinks that η is distributed by $g(\eta)$. We also presume, for simplicity, that our producer has an accurate perception of the mean price:

$$E_g[P_X(\eta)] = \int_\eta P_X(\eta)g(\eta)d\eta = B' .$$

We can now show that if the output decisions at the X-firm must be made before the true value of η is observed, then the output response to any delivery of Q is precisely the one noted above for the socially motivated producer. Assuming accurate knowledge of the mean price can also be shown to be costless in terms of economic content. Output under both types of control would certainly be suboptimal were it violated, but the overall effect on the comparison is entirely neutral.

Were the center to issue a quantity order to the Q-firm, the K-response of the X-firm that maximized expected profits is implicitly defined by the first order condition that

$$E_g\left\{P_X(\eta)\left[\frac{(\gamma[\hat{K}(Q)]^\rho + (1-\gamma)Q^\rho)^{1/\rho}}{\hat{K}(Q)}\right]^{1-\rho}\right\} = r\,.$$

Equality is guaranteed by (6a′) if

$$\hat{K}(Q) = Z^\sigma Q. \tag{13}$$

Incorporating (13) into the center's maximization subsequently reveals that $\hat{Q}_0$ remains the best quota; $\hat{K}_0$ is then employed and $\hat{X}_0$ produced.

Given a price order, the Q-firm still produces $[\hat{Q}_0 + h(p, \theta)]$. The X-firm then responds by employing K up until

$$E_g\left\{P_X(\eta)\left[\frac{(\gamma[\hat{K}(h)]^\rho + (1-\gamma)[\hat{Q}_0 + h(p, \theta)]^\rho)^{1/\rho}}{\hat{K}(h)}\right]^{1-\rho}\right\} = r\,. \tag{14}$$

The K-response that solves (14) is simply

$$\hat{K}(h) = Z^\sigma[\hat{Q}_0 + h(p, \theta)]\,. \tag{15}$$

Viewing the center's maximization in the light of (15) now reveals that $\tilde{p} = C'$ is still the optimal price. Deliveries of Q, employment of K, and output of X are therefore unchanged for all θ.

The profitability problem that exists under price control with expected marginal value pricing inherits a second dimension in this case; the subjective expected profitability of the X-firm is now the crucial concern. We can infer from the last paragraph, however, that the sign of expected profits as seen by the producer of X depends on the subjective covariance of $P_X(\eta)$ and $\tilde{X}(\theta) = A(\rho)\tilde{Q}(\theta)$. Were the producer to feel that this covariance is negative, he would expect a loss and experience pressure to avoid the control of Q. It should be clear, despite this complication, that the input pricing scheme outlined toward the end of Section 3 will solve not only the problem of actual expected profits delineated there, but also this problem of subjective expected profits.

5. OUTPUT DISCREPANCY UNDER QUANTITY CONTROL

We have thus far ignored the possibility that the producer of an intermediate good may be unable to fulfill the prescribed quantity order exactly. To correct this omission, we now incorporate a random output discrepancy under the quantity mode. The quantity delivered to the X-firm, Q_d, is assumed to be additively related to the quantity ordered by the center, Q_p:

$$Q_d = Q_p + \phi(\xi)\,.$$

The vector ξ is meant to reflect any production or motivational effects that could cause the Q-firm to undershoot or overshoot the targeted output. The cost function should also reflect the addition of this distortion, since many of these effects are cost related. We represent such effects in the cost approximation, with some abuse of notation, by dichotomizing the original θ:

$$C(Q, \theta, \xi) = a(\theta, \xi) + [C' + \alpha(\theta, \xi)](Q - \hat{Q}_0) + 1/2C_{11}(Q - \hat{Q}_0)^2 .$$

We need only concern ourselves with a change in the optimal quantity order, since $\phi(\xi)$ effects only the quantity mode. The optimal price control remains C' under which the output response of the X-firm is still $\tilde{X}(\theta, \xi) = A(\rho)\tilde{Q}(\theta, \xi)$.

The center determines the optimal quantity order, $\hat{Q}_p$, by maximizing expected benefits minus expected costs with respect to Q_p. The X-firm meanwhile selects its K response to a delivery of $[\hat{Q}_p + \phi(\xi)]$ by maximizing expected social benefits minus the private input costs. When we assert that

$$\hat{Q}_p = [\hat{Q}_0 - E\phi(\xi)] . \tag{16}$$

the first order conditions of these maximizations reduce to

$$B'[A(\rho)]^{1-\rho} = C'$$

$$B'[A(\rho)/Z^{\sigma}]^{1-\rho} = r$$

respectively. Equations (6) guarantee the veracity of these equalities, while the assumed shapes of the benefit and cost schedules guarantee the uniqueness of (16) as a solution. The reader should note that this analysis is valid under any of the previously recorded pricing schemes. An amount $\hat{X}(\xi) = A(\rho)\hat{Q}_d(\xi)$ is produced as $\hat{K}(\xi) = Z^{\sigma}\hat{Q}_d(\xi)$ is employed with the delivered $\hat{Q}_d(\xi) = \hat{Q}_p + \phi(\xi)$.

6. THE COMPARATIVE ADVANTAGE OF PRICE CONTROLS

We can now compare price and quantity control of Q in the context of the output distortion that we have just introduced. Independent of the behavioral and pricing assumptions that are explored above, an amount

$$\hat{Q}_d = \hat{Q}_p + \phi(\xi)$$

is delivered to the X-firm under the optimal quantity control, and

$$\hat{X}(\xi) = A(\rho)\hat{Q}_d(\xi)$$

is produced. Optimal control by $\tilde{p} = C'$ results in deliveries of

$$\tilde{Q}(\theta, \xi) = \hat{Q}_0 - [\alpha(\theta, \xi)/C_{11}]$$

and a final output of

$$\tilde{X}(\theta, \xi) = A(\rho)\tilde{Q}(\theta, \xi) .$$

The comparative advantage of prices over quantities is therefore

$$\begin{aligned} \Delta(\rho) \equiv\ & E\{B[\tilde{X}(\theta, \xi), \eta] - r\tilde{K}(\theta, \xi) - C[\tilde{Q}(\theta, \xi), \theta, \xi]\} \\ & - E\{B[\hat{X}(\xi), \eta] - r\hat{K}(\xi) - C[\hat{Q}_d(\xi), \theta, \xi]\} \\ =\ & 1/2B_{11}\{\mathrm{Var}[\tilde{X}(\theta, \xi)] - \mathrm{Var}[\hat{X}(\xi)]\} - \mathrm{Cov}[\beta(\eta); \hat{X}(\xi)] \end{aligned} \tag{17}$$

$$+ \operatorname{Cov}[\beta(\eta); \tilde{X}(\theta, \xi)] + 1/2C_{11} \operatorname{Var}[\tilde{Q}(\theta, \xi)]$$
$$+ 1/2C_{11} \operatorname{Var}[\hat{Q}_d(\xi)] - \operatorname{Cov}[\alpha(\theta, \xi); \hat{Q}_d(\xi)].$$

Each of the expressions in (17) can be easily interpreted.

There are two effects on the benefit side. Variation in the production of X will cause expected benefits to fall below the level that would be achieved were the mean output ($\hat{X}_0$) produced with certainty. This loss increases both as the curvature of the benefit function increases ($|B_{11}|$) and the variance of output increases; the first term in (17) captures this effect. The control that creates the smaller variance will therefore receive a positive bias in the comparison for any nonlinear benefit schedule. A secondary effect can occur when η and (θ, ξ) are not independent, and is reflected in the second and third terms. If $\operatorname{Cov}[\beta(\eta); \hat{X}(\xi)]$ is positive, for instance, the production of X under quantity control increases, on the average, as the marginal benefit schedule shifts upward. This being the correct direction, a positive bias toward quantities (a negative bias against prices) should be recorded; the covariance is therefore subtracted in the comparative advantage. A similar explanation justifies the subsequent addition of $\operatorname{Cov}[\beta(\eta); \tilde{X}(\theta, \xi)]$.

Similar influences are felt on the cost side. Variation in the production of Q will cause expected costs to rise above the level that would be achieved were the mean output ($\hat{Q}_0$) produced with certainty. The fifth expression in (17) records the impact of this loss under quantity control on the comparative advantage. The sign of the final covariance indicates whether the production of Q under quantities moves correctly with respect to the randomly shifting marginal cost schedule. Because the Q-firm maximizes actual profits when faced with a price order, however, output will always move correctly under prices. This is precisely the efficiency gain of having price equal to actual marginal cost for all (θ, ξ) and can be shown to outweigh the increase in expected costs:

$$\{-1/2C_{11} \operatorname{Var}\tilde{Q}(\theta, \xi) - \operatorname{Cov}[\alpha(\theta, \xi); \tilde{Q}(\theta, \xi)]\} = 1/2C_{11} \operatorname{Var}\tilde{Q}(\theta, \xi).$$

The fourth term in (17) therefore reflects both effects, and is always positive.

It is possible to express (17) entirely in terms of Q:

$$(18)\quad \Delta(\rho) = 1/2B_{11}[A(\rho)]^2[\operatorname{Var}\tilde{Q} - \operatorname{Var}\hat{Q}_d] + [A(\rho)]\operatorname{Cov}[\beta(\eta); \hat{Q}_d - \tilde{Q}]$$
$$+ 1/2C_{11}[\operatorname{Var}\tilde{Q} + \operatorname{Var}\hat{Q}_d] - \operatorname{Cov}[\alpha(\theta, \xi); \hat{Q}_d].$$

The elasticity of substitution between K and Q appears only in the multiplicative factor $A(\rho)$, the same factor that translates deliveries of the intermediate good into production of the final good. Equation (18) thereby implies strongly that the elasticity of substitution simply determines the importance of the benefit side to the comparison. The two extreme cases with which we motivated this discussion provide a perfect setting in which to begin to substantiate this secondary interpretation.

We argued in Section 2 that when K and Q are perfect subsitutes, the output of X will remain constant even as deliveries of Q vary. This conclusion is true, of course, regardless of the source of the variation in Q, and thus, regardless of the type of control placed on Q. The comparative advantage of prices would therefore be totally void of a benefit side:

$$\Delta(1) = 1/2C_{11}\{\mathrm{Var}[\tilde{Q}(\theta, \xi)] + \mathrm{Var}[\hat{Q}_d(\xi)]\} - \mathrm{Cov}[\alpha(\theta, \xi); \hat{Q}_d(\xi)].$$

Only the last term can be negative, and that only when the marginal cost schedule and the output distortion are positively correlated. The first term registers the always positive net bias of output variation under prices. There is no counter-balancing efficiency gain under quantities, however, so that the second variance term, the increase in expected costs due to output variation under quantities, is also positive. $\Delta(1)$ is therefore quite likely to be positive, especially when C_{11} is large. For our present purposes, however, this observation is overshadowed by the result that infinite substitutability has caused the benefit side of the comparative advantage to disappear entirely.

The fixed coefficients case was also noted in Section 2; recall that when the elasticity of substitution is zero, $X=(1-\gamma)Q$. The comparative advantage of prices under these circumstances is

$$\Delta(\infty) = 1/2B_{11}(1-\gamma)^2[\mathrm{Var}\,\tilde{Q} - \mathrm{Var}\,\hat{Q}_d] + (1-\gamma)\mathrm{Cov}[\beta(\eta); \hat{Q}_d - \tilde{Q}]$$
$$+ 1/2C_{11}[\mathrm{Var}\,\tilde{Q} + \mathrm{Var}\,\hat{Q}_d] - \mathrm{Cov}[\alpha(\theta, \xi); \hat{Q}_d].$$

The benefit side has been modified by powers of $(1-\gamma)$, and since $(1-\gamma)<1$, its importance is still diminished. Little can be said about the sign of $\Delta(\infty)$.

The difficulty with testing our interpretation through the intermediate cases lies not in the determination of the comparative advantage of prices, but rather in the determination of the impact on $\Delta(\rho)$ of a change in the elasticity of substitution. All of the previous analysis is valid for an arbitrary ρ, and thus an arbitrary $\sigma(\rho)$, but requires that the value of ρ remain fixed once it has been chosen. To emphasize the fact that the points around which the approximations are made depend crucially upon that initial value, we now designate it by ρ_0. In addition and for the sake of comparison, we will be changing the elasticity in such a way that the socially optimal level of Q is maintained at $\hat{Q}_0$. The K-response of the X-firm to a delivery of $\hat{Q}_0$ is then simply

$$\hat{K}_\rho = Z^{\sigma(\rho)}\hat{Q}_0$$

for any ρ and $\sigma(\rho)=[1/(1-\rho)]$, and the resulting production of X

$$\hat{X}_\rho = [\gamma Z^{\rho\sigma(\rho)} + (1-\gamma)]^{1/\rho}\hat{Q}_0 \equiv D(\rho)\hat{Q}_0.$$

The optimal price order emerges from this complication intact, and elicits in the following responses:

$$\tilde{Q}(\theta, \xi) = \hat{Q}_0 - [\alpha(\theta, \xi)/C_{11}],$$
$$\tilde{K}_\rho(\theta, \xi) = Z^{\sigma(\rho)}\tilde{Q}(\theta, \xi), \quad \text{and}$$
$$\tilde{X}_\rho(\theta, \xi) = D(\rho)\tilde{Q}(\theta, \xi).$$

The optimal quantity is similarly unchanged, so that

$$\hat{Q}_d(\xi) = \hat{Q}_p + \phi(\xi),$$
$$\hat{K}_p(\xi) = Z^{\sigma(\rho)}\hat{Q}_d(\xi), \quad \text{and}$$
$$\hat{X}_p(\xi) = D(\rho)\hat{Q}_d(\xi).$$

The comparative advantage can now be expressed in exactly the same form as before, but it stands valid for any elasticity of substitution in the neighborhood of $\sigma(\rho_0)$:

$$(19) \quad \Delta(\rho/\rho_0) = 1/2B_{11}[D(\rho)]^2[\operatorname{Var}\tilde{Q} - \operatorname{Var}\hat{Q}_d] + D(\rho)\{\operatorname{Cov}[\beta(\eta); (\hat{Q}_d - \tilde{Q})]\} + 1/2C_{11}[\operatorname{Var}\tilde{Q} + \operatorname{Var}\hat{Q}_d] - \operatorname{Cov}[\alpha(\theta, \xi); \hat{Q}_d].$$

All that remains is to compute the effect on $D(\rho)$ of a change in ρ; it is easy to note that

$$(20) \qquad \frac{\delta D(\rho)}{\delta \rho} = -[\rho(1-\rho)]^{-2}D(\rho)\gamma Z^{\rho\sigma(\rho)}[\ln D(\rho)][\ln Z].$$

The sign of (20), and thus the direction of the effect of a change in ρ, clearly turns with the signs of the logarithmic terms. There are four possible cases depending upon whether Z and ρ are positive or negative. Considering each case individually reveals that

$$(21) \qquad \frac{\delta D(\rho)}{\delta \rho}\begin{cases} <0 & 1>\rho>0 \\ >0 & 0>\rho. \end{cases}$$

We can see from (21) that when ρ is positive, an increase in the elasticity of substitution will cause a decrease in the $D(\rho)$ coefficient. The output effects of variation in the deliveries of the intermediate good diminish in magnitude. If we recall that the cost side of the comparison tends to favor prices and view this influence as a decrease in the importance of the benefit side, we can argue that increased substitutability favors price control when ρ is already positive. The opposite conclusion is drawn when ρ is negative. The output effects of variation in the deliveries increase from a factor of $(1-\gamma)$ when K and Q are employed in fixed proportions. The importance of the benefit side increases as ρ climbs through its negative range, and the K response at the X-firm continues to accentuate the effect of variance in Q.

The output variation for a specific variance in deliveries reaches a maximum in the Cobb-Douglas case in which $X = Z^{\gamma}Q$. Variation in delivery of the intermediate good is therefore exaggerated in the variation of X when $\gamma C' > (1-\gamma)r$, reduced when $\gamma C' < (1-\gamma)r$, and transferred intact when equality holds. We see immediately that it is entirely possible for variation in the output of the final good never to exceed that of the intermediate good, regardless of the elasticity of substitution in the process that produces X from Q.

Before we close, there are a couple of loose ends that need to be tied. The first concerns a profit maximizing producer of X with an inaccurate perception of the mean price. His responses to a given delivery of Q will certainly reflect his misinformation, but the comparative advantage emerges in the same general form as Equation (19). The only crucial difference is that this X-firm produces a distribution of X around the wrong mean. The variance and covariance terms in (19) that involve X therefore become simply the corresponding moments around that incorrectly computed mean output level for the final good.

We should also record a few observations on the role of inventories in our comparison. Their influence, quite obviously, would be to reduce the variation in the amount of Q delivered to the producer of X, and thereby reduce the variation in the output of X, itself. Since $E\tilde{Q}(\theta, \xi)=E\hat{Q}_d(\xi)=\hat{Q}_0$, it would be possible to maintain a store of Q so that $\hat{Q}_0$ could be delivered in all states of nature. In this extreme case, $\hat{X}_0$ would always be produced with an expected profit of zero, even when the expected marginal value product is charged for Q. The maintenance of inventories implies, of course, a dead weight loss in foregone consumption, and it is unlikely that levels sufficient to guarantee constant delivery of $\hat{Q}_0$ would be optimal. To the extent that inventories at any positive level diminish the variation in the deliveries of Q, however, they diminish the importance of the benefit side of the comparison. As a rule, therefore, price controls should be more preferred, or quantities less, as the level of inventories in the intermediate good increases.

7. SOME CONCLUDING REMARKS

We have demonstrated that the elasticity of substitution affects the degree with which the variation in the output of the intermediate good is translated into variation in the subsequent production of the final good. An increase in that elasticity will, for example, increase or decrease the importance of the benefit side of the prices-quantities comparison as σ is less than or greater than one. Maximum translation of variation in delivery therefore occurs in the Cobb-Douglas case in which σ is precisely one. Inasmuch as the cost side tends to favor prices because output under prices moves in the correct direction relative to marginal costs, deviation from Cobb-Douglas favors price control.

The profitability of producing the final good was a second concern, since negative expected profits would create pressure to avoid regulation by integrating the production of the intermediate good into the production of the final good. Pricing policies that both eliminate this difficulty, and leave the behavior of the X-firm otherwise unaffected were demonstrated. We should also note in closing that it is possible to construct such schemes for a profit maximizing producer of the final good who works with an inaccurate subjective distribution of the *ex post* selling price of that good. The role of inventories was also noted briefly. To the degreee that positive inventories lessen the variation in input deliveries, the importance of the benefit side of the comparative advantage is diminished and prices are more favored.

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