CARBON EMISSIONS TAXES: Their Comparative Advantage Under Uncertainty

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0. INTRODUCTION

This paper was, at its inception in the fall of 1990, intended to review a growing literature on carbon taxes—taxes that have been proposed as one policy option that would be available to the international community if it were to choose to slow the rate of greenhouse-induced global warming by curbing the worldwide emission of carbon dioxide. The original idea was to compare and contrast published answers to a wide range of important questions. What levels of taxation would, for example, be required to achieve specific emissions reductions? How much aggregate economic activity would be sacrificed as a result? How would these taxes and their associated efficiency losses evolve over time if the specified emissions reduction were to be permanent? How would the answers to these and other questions be altered by adopting alternative baseline scenarios of what an unregulated future might hold? How
sensitive are these answers to alternative assumptions about where and how the enormous tax revenues might be distributed and then spent?

These and other questions are certainly raised by the very suggestion of taxing carbon emissions, but this paper will not tend to a review of their consideration. A number of other authors had already undertaken that task by the fall of 1991, when this manuscript was due to be delivered to the editors, and the fruits of their labors will certainly have seen the light of day prior to the release of the present publication in the fall of 1992.¹ The results of Energy Modeling Forum–12 (EMF-12), a systematic comparative research effort designed and coordinated by Darius Gaskins and John Weyant, will also be published in the early summer of 1992; it will provide a direct vehicle for comparing the answers to many of the questions noted above across 14 separate energy models.² Any review drafted at this juncture, early in 1992, would therefore be unquestionably superfluous at best, and most probably hopelessly outdated and incomplete even on the first day that it appeared. The original idea has, therefore, been discarded.

In its place is a preliminary discussion of the scope of the next set of questions—questions about how to design a regulatory mechanism to achieve specific long-term targets under conditions of enormous uncertainty. Assuming that some reduction in carbon emissions is desirable, analytical attention should be focused upon determining how best to effect that reduction. Some uncertainty will remain over the long term, and its resolution will affect the evolution of the regulatory targets over time independent of policy design. Other uncertainty is more short term, and will define how individuals, markets, governments, and other institutions respond to the regulatory environment; and it is here that questions of policy design become important. Should a carbon tax be employed with its incumbent welfare loss, or does there exist a more efficient regulatory alternative that can achieve the same long-term goal while minimizing short-term efficiency losses?

Section 1 and Appendix A present a simple theoretical structure within which the major alternatives can be portrayed and their relative merits explored. The model described there is derived directly from the framework first offered by Weitzman (8) in 1974; it was expanded and applied to

¹See, for example, Boero, Clark, and Winters (1), Cline (2, 3), Darmstadter & Plantiga (4), Edmonds & Barns (5), and/or Hoeller, Dean, and Nicolaisen (6)—a small sample of what was available at the end of 1991.

²See Gaskins & Weyant (7) for the final report summarizing the results of their comparative work. Each of the 14 models investigated a wide range of emissions reduction strategies along common, baseline scenarios of what the next century might hold in store. Many working papers and draft reports are cited there. Some provide more detailed descriptions of the workings of the various energy models: aggregate, global, and national “top-down” models; detailed sectoral, econometric models; and even more detailed, thoroughly disaggregated “bottom-up” models. Others provide detailed comparisons of the modeling structures and their underlying assumptions.
environmental issues by Spence & Roberts (9) in 1978 and by Yohe (10, 11) in 1978 and 1979. Fundamentally, applying the results of these comparisons of alternative regulatory strategies under conditions of uncertainty to the control of carbon emissions suggests that variation in total emissions that would be allowed by each alternative is critical in determining their relative merit. Simply put, a carbon tax would allow total emissions to vary from year to year in response to unforeseeable changes in the derived market demand for fossil fuel even as average total emissions move over time toward a specific target. This variation would reduce the expected cost of the control, on the demand side of the problem, but it would also increase the expected social damage of resulting emissions. A critical trade-off between efficiency on the control side and increased expense on the damage side is thereby identified.

Section 2 explores the variability in emissions for a specific taxation strategy—one that can be expected to reduce carbon emissions linearly to 80% of their 1990 levels by the year 2010. A simple version of an economically consistent aggregate global energy market is, to that end, fully developed in Appendix B. Straightforward initialization of its driving parameters along trajectories consistent with current wisdom of their most likely futures is seen to support an unregulated baseline scenario that is itself consistent with current expectations of fossil fuel consumption and carbon emissions. Simulations of possible variation around this best-guess baseline are thereby produced, and they support a reasonable representation of the uncertainty within which researchers can presently quantify a baseline—a workable measure of the variability in total emissions upon which the policy design issue is shown to rest in Section 1.

The discussions presented in Sections 1 and 2 may appear technical and out of place. After all, they review and synthesize two theoretical constructions rather than survey a large body of literature. They are, however, firmly rooted in the first principles of economic theory—principles that must be clearly understood before correct decisions about the design of global carbon emissions controls can be made. A concluding section adds some perspective to their application by highlighting them one final time. It also argues that clearly understanding the design trade-off identifies the potential for yet another source of disagreement between developed countries and the rest of the world on how to manage global environmental issues. Perhaps most importantly, it thereby offers a preview of a new set of questions that (a) are derived from the comparison of alternative regulatory strategies, (b) must be answered before any regulatory decision is made with any degree of comfort.

This strategy is one of the many options considered in the EMF-12 work. It was chosen there, and here, because it clearly articulates how one of the intermediate emissions reduction targets currently being offered to the United Nations by many members of the world community might be enforced.
that an opportunity to minimize welfare cost has not been missed, and (c) probably will not have been answered by the end of 1992.

1. THE COMPARATIVE ADVANTAGE OF A CARBON TAX

A regulatory authority that is trying to administer the production or the consumption of some specific product or some particular factor of production has a wide assortment of regulatory tools at its disposal. It might choose to design a price mechanism, for example, setting a price (or adding a tax or subsidy to an existing price) and allowing everyone involved the autonomy to decide how much to offer for sale, how much to purchase, or how much to employ. This is the carbon tax alternative for reducing the worldwide carbon emissions—add a constant or growing tax to the price of fossil fuel (contingent on its carbon content) and let the future run its course, administering "mid-course" corrections along the way as the future unfolds.

Alternatively, the authority might choose to design a set of quantity standards, allocating specific quantities to each and every producer, consumer, or employer; it would thereby be assured of knowing exactly who would produce, consume, or employ what percentage of some predetermined total. This corresponds to fixing carbon emissions standards for each and every country or region around the globe, again altering the total allocation and its distribution from time to time as needed to accommodate new knowledge of what is actually happening.

An authority might also choose to design a marketable permit system, allocating licenses or permits that would entitle their owners to produce, consume, or employ specified quantities of the regulated product or factor and allowing them to buy or sell those permits at their own discretion. It would, in that case, be assured of knowing total production, consumption, or employment exactly (it would equal the sum of the permits issued), but it would not know which producer, consumer, or employer would produce, consume, or employ how much of that total. The fixed regional distribution of carbon emissions associated with the fixed standards regime would be relaxed, under this alternative, but total global emissions would be fixed at a prescribed level—a level that the authority could adjust in the future with or without consideration of distributional issues. The question of choosing

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4 The discussion presented here and in Appendix A draws heavily upon earlier work by Weitzman (8) and Yohe (9, 10). Extensions of the general model appear in Laffont (12) and Karp & Yohe (13). Similar models have been applied successfully in other arenas: by Poole (14) to questions of how to design instruments of monetary policy and by Pelcovits (15) and Stiglitz & Dasgupta (16) to questions of when to favor quotas over tariffs and visa versa.

5 Distributional issues are, of course, critical in the overall scheme of things. They do not, however, influence necessarily the choice of the best regulatory policy design. Any allocation of emissions or revenue can be accomplished by any of the design options being considered.
among these alternative policy designs is, then, one of determining the relative efficiency of these competing designs under a variety of circumstances.

1.1. Regulatory Alternatives in a Certain Environment

A world of perfect certainty would allow an authority facing these alternatives the security of an important and well-known equivalence result: there exists, for any targeted level of total production, consumption, or employment, an equivalent price control that supports the appropriate efficient allocation of that total across the relevant set of individuals. This is simply a statement of the textbook equivalence of tariffs and quotas. Marketable permits in a world of perfect certainty represent only a slight wrinkle on the quantity control theme. Their marketability allows the efficient allocation of production, consumption, or employment to be achieved across all individuals regardless of the initial distribution of specific quantity standards. Individuals finding that they have more permits than they need, because they plan to produce, consume, or employ less than their allocated standard, would simply sell their excess supply to individuals who find themselves in the opposite situation. The efficient allocation supported by the equivalent price control results because the permit market clears at precisely that price. The permit scheme could even generate the same revenue for the authority if it were to allocate all of the permits to itself and auction the full supply in a competitive market.

A simple model that links the consumption of fossil fuel to potentially damaging carbon emissions—the problem that motivates this paper—can quickly illustrate this equivalence. Let the derived demand (or marginal benefit—see below) curves for fossil fuel from two separate firms (or two countries or regions in a global policy environment) be given by $\text{MB}_i = P_d = a - bF_i$, where $F_i (i = 1, 2)$ represents consumption of fossil fuel $F$ by firm $i$, $P_d$ records the common price facing both demanders, and $a$ and $b$ are non-negative constants. The resulting market demand for fossil fuel $F$ would then be $\text{MB} = P_d = a - [b/2]F$, where $F = F_1 + F_2$. Both of the original derived demand curves can be thought to represent firm-specific marginal benefit schedules [denoted $\text{MB}_i$], so the aggregate derived demand curve can be interpreted as an aggregate marginal benefit curve for the consumption of fossil fuel, $\text{MB}$.

Turning now to the cost side of the market for fossil fuel, let $c_0$ represent the marginal private cost of using fossil fuel. Supplement this constant marginal private cost with a marginal social cost increment given by $\text{MC}(F) = c_1F$, where $c_1$ is the rate at which marginal social costs of emissions

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6There is no reason to confine our attention to only two firms with identical, linear-derived demand schedules. The results reported here are sufficiently general to accommodate relaxing any of these simplifying assumptions. Only clarity of the exposition and its extension to uncertain demand in the next subsection would suffer if they were, in fact, relaxed.
increase as a function of their source—the consumption of fossil fuel. It is through the parameter $c_1$ that technology and variation in the mix of fossil fuels actually used enter the analysis. The socially optimal level of fossil fuel emission, $F^*$, can now be characterized as one that sets marginal social cost, $MC(F) = c_0 + c_1 F$, equal to aggregate marginal benefits; i.e.

$$F^* = \frac{2(a - c_0)}{(2c_i d + b)}.$$ 

Each firm would then produce

$$F_i^* = \frac{(a - c_0)}{(2c_i d + b)} \quad (i = 1, 2)$$

in support of this total, and the efficient market clearing price for fossil fuel would emerge from the aggregate demand curve as

$$P^* = a - \left[\frac{b(a - c_0)}{(2c_i d + b)}\right]$$

Faced with the problem of regulating consumption to accommodate its marginal social cost, a regulator in a world of certainty could choose one of three options:

1. specify the $F_i^*$ given in equation 2;
2. add a tax $t^*$ to the private marginal cost price $c_0$, where

$$t^* = [P^* - c_0] = \frac{2c_i d (a - c_0)}{(2c_i d + b)}$$

3. arbitrarily allocate $F^*$ marketable permits, each allowing the eventual holder to consume one unit of fossil fuel.

In the last case, the permit market would clear at $t^*$, and the equilibrium allocation of consumption would match the $F_i^*$ of equation 2.8

1.2. Regulatory Alternatives in an Uncertain Environment

The world of a regulatory authority is not, however, one of perfect certainty, particularly when the unknown long-term prospects of global change are being considered. The only certainty in that regulatory environment is, in fact, that any policy specification will turn out to be wrong when the future actually unfolds. Any policy will, therefore, carry the baggage of excess welfare cost, and the key in choosing between regulatory options must therefore hinge upon

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7 This is a long-term damage function that accounts for the present value of current and future damage attributed to each unit of emissions.

8 See Yohe (10) for a more complete description of why this equivalence holds in a world of perfect information.
minimizing that loss. A policy choice must, in other words, weigh the welfare gain allowed by efficient reaction to future events against the expected welfare cost of that flexibility even as the overall target of the regulation changes over time.

In the emissions problem just modeled, for example, uncertainty about year-to-year variability in the structure of future individual derived demand curves for fossil fuel would make it necessary to compute alternative regulatory specifications on the basis of an expected aggregation of annual demand. A set of marketable permits based upon those computations would fix total annual carbon emissions at some predetermined level, would allow efficient allocation of the corresponding limit on fossil fuel across potential demanders, but could easily cause the shadow price of fossil fuel (the price of the permits) to grossly underestimate or overestimate the actual marginal social cost of emissions. An efficiency loss would thereby result. A certainty equivalent tax, on the other hand, would fix the shadow price of total annual emissions, but not necessarily at the appropriate level. The result could be consumption and emissions that are dramatically too high, or too low, and another, different efficiency loss would be recorded.

To see how these errors might affect the relative merits of the two primary alternatives, suppose that the derived demands for fossil fuel for the two firms of the previous model were given by \( MB_i = P = a + x_i - bF_i \), where the \( x_i \) are random variables. Assume, without loss of generality, that the \( x_i \) have means equal to zero so that the original intercepts incorporate the current view of its expected value. If the authority charged with limiting carbon emissions must specify a regulatory environment before the true values of the \( x_i \) are known, however, then the appropriate social objective is to maximize expected benefits minus costs. The assumed structure of the \( x_i \) means, however, that this expected value maximization problem has already been solved. Total consumption equal to \( F^* \) specified in equation 1 should be the targeted aggregate quantity, and marketable permits would allow that quantity to be allocated across the two firms most efficiently regardless of the values assumed by \( x_1 \) and/or \( x_2 \). The resulting fixed-level emissions would fix external costs, but the corresponding fixed level of consumption could be too high or too

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9 A certainty equivalent tax achieves an expected level of emissions equal to the appropriate targeted level; emissions may be too high or too low, in fact, but these errors would average to zero with repeated trials.

10 It is assumed, in what follows, that the problem of choosing targets that maximize expected social welfare has been solved correctly. That assumption is, however, not essential when it comes to comparing alternative control strategies designed to achieve, at least in expected value, the same targets. The key is to compare the marketable permit option with its certainty equivalent tax—the tax for which expected total emissions equal the total number of unit permits to be distributed.
low; there could, as a result, be a significant welfare cost on the consumption side of the market. By way of contrast, the tax that maximizes expected net benefits is given by $t^*$ from equation 4. Expected consumption would then equal $F^*$, but actual total consumption and emissions would depend upon the values assumed by $x_1$ and/or $x_2$. Consumption would respond efficiently to the actual state of aggregate demand, creating a welfare gain relative to total quantity restriction of the permit scheme; but emissions would be uncertain and the expected social cost of their potential damage would rise. There is, therefore, a trade-off to be considered.

Appendix A records a detailed exploration of this trade-off. It ends with a computation of a critical "comparative advantage" statistic—the difference between the expected dead weight loss associated with a marketable permit scheme and the comparable expected dead weight loss associated with the certainty equivalent tax:

$$Z = E\{DWL(\text{per})\} - E\{DWL(t^*)\} = [(b - 2c_1)/4]\{\text{Var}[\text{FT}(X)]\}.$$  

Notice that this comparative advantage is negative, indicating a preference toward the marketable permits, whenever the aggregate marginal benefit schedule is less steeply sloped at $F^*$ than the marginal social cost curve. In the extreme, in fact, $Z$ becomes arbitrarily negative as $c_1$ grows arbitrarily large. Why? Steeply rising marginal social costs (i.e. high values for $c_1$) simply mean that the social cost curve is sharply concave in the neighborhood of the targeted level of emissions. Expected social costs therefore rise quickly when any variation in total consumption (and thus emissions) is allowed, and the regulatory scheme that discourages that sort of variation—the permit scheme—should be strongly preferred.

The comparative advantage statistic of equation 5 also urges caution in allowing flexible, stochastic response to the regulatory environment whenever that response grows large. If the aggregate derived demand for fossil fuel were highly elastic in the neighborhood of $F^*$, for example, then the slope parameter $b$ would be relatively small. Taking this case to the extreme by allowing $b$ to

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11 If, for example, both random variables were positive, then the actual aggregate marginal benefit curve would lie above the expected aggregate marginal benefit curve, and $F^*$ would be suboptimally low with marginal benefit exceeding marginal cost. If both were negative, of course, the opposite would occur.

12 In the case where both variables were positive, to continue the example more completely described in footnote 8 above, the consumption of fossil fuel at both firms in response to the certainty equivalent tax would be greater than the $F_t^*$ given in equation 2—so much greater that total consumption would be suboptimally high in this case and marginal cost would exceed marginal benefit.

13 This is equation A.7 from Appendix A.
fall toward zero, in fact, again suggests the likelihood of a negative comparative advantage and a preference toward the permit mechanism that prohibits variation in consumption and emission. Why, this time? Low values for the slope of aggregate demand mean that even small stochastic variation in the intercept translates into large variation in consumption and emissions given the tax. Even if expected social costs were even moderately concave, in this case, emissions variability would be so large that it could not be tolerated.

It must be emphasized, however, that the variation allowed by the tax alternative is, in and of itself, the reflection of efficient reactions to stochastic changes on the demand side of the market—changes that improve expected net welfare that the authority cannot foresee as it makes its decision on how to achieve its regulatory target. Undertaking the design of a regulatory mechanism that would allow this sort of demand-driven variation in consumption and emission should therefore carefully be considered. It is simply the lesson of equation 5 that a policy that explicitly encourages flexibility should be adopted only when the associated increment in expected social cost is not too extreme. That is why equation 5 indicates that the sign of the comparative advantage changes so that the tax scheme that encourages this efficiency should be preferred to a marketable permit scheme that fixes total emissions only when marginal social costs are less steeply sloped than aggregate marginal benefits—i.e. only when the efficiency gains of flexibility exceed the associated welfare losses on the cost side.

This entire discussion of the sign notwithstanding, the magnitude of the comparative advantage statistic, and thus the importance of the policy design choice, depends on the size of the variance of total consumption and total emissions under the tax alternative. That variance is rooted, from the perspective of the regulatory authority, in the uncertainty with which derived demand is viewed; but it is displayed functionally and can be observed directly in terms of variability in total consumption and total emissions. Since the variance of total emissions is, itself, a variance of a sum of random variables, there are covariance terms across the various sources of demand to be considered.

If the derived demands for fossil fuel were independently distributed across the two (many) firms (or countries), for example, then the covariances would all be equal to zero and the variance of the sum would be the sum of the

\[ \text{Variance of total fossil fuel consumption} = \frac{\text{Variance in the sum of the stochastic intercept disturbances}}{b^2} \]

The comparative advantage statistic therefore becomes arbitrarily negative as \( b \) approaches zero.
individual variances. If variation in derived demand were negatively correlated, however, then some natural cancellation would occur as periods of high demand for one (some) firm would tend to be matched with periods of low demand at the other(s); the variance of the total would then be diminished by negative covariances, and the importance of choosing correctly between the two alternative regulatory schemes would decline. If variation in derived demand were positively correlated, though, as would be the case for interdependent world markets working through internationally synchronized business cycles, then some natural amplification of individual variation would occur in the variance of the sum. The variance of total consumption would be higher, as a result, and the choice between regulatory alternatives would become more important.

In any case, variation in the total consumption of fossil fuel, and the corresponding level of carbon emissions, has been seen to be a critical statistic in weighing the importance of choosing correctly between some sort of global carbon tax and system of marketable emissions permits. Does the choice matter, or is this really a non-issue disguised as a quandary? The next section will explore the determinants of this variation as a first step toward addressing these questions of context. Before turning to those questions, however, it is important to address all of the uncertainty on the social cost side of the problem, which has been ignored here. There are, of course, enormous uncertainties on the impacts side, and they should be incorporated into the construction of the marginal social cost of fossil fuel consumption and carbon emission. Some of these uncertainties have been examined and quantified, but others have simply been identified. Still others have yet to be discovered.15 The analysis presented above does not ignore them because they are unimportant or because they have not yet been quantified. Rather, they are ignored in an analysis of what type of control to impose because improved understanding of what they are and what value they will assume will not affect how the demanders of fossil fuel will respond to a particular regulatory environment. Improved understanding of their sources and importance will certainly cause regulators to change the targets of their activity as the future unfolds, but improved understanding on the cost side will not directly influence questions of policy design driven by how people respond to regulatory environments constructed to meet those targets. Put another way, questions of which type of policy to design and install will persist even as the regulators adjust their targets.

15See Table 1 in Scott et al (17) for a catalog of potentially vulnerable sectors (ranging from unmanaged ecosystems through agriculture and seaside real estate and into human health), a description of what is currently known about the physical impacts of global change that place those sectors in danger, and a comparable description of what is still currently uncertain about those impacts and their economic consequences.
2. EMISSIONS VARIABILITY\textsuperscript{16}

The fundamental results of Section 1 suggest that the comparative advantage of a carbon tax over a marketable permit scheme that fixes total emissions (the $Z$ statistic of equation 5), the resulting variability of the market permit price, and ultimately the importance of choosing the correct policy design all depend upon the variance of total emissions that would be allowed by the tax. A prudent evaluation of the relative merits of the two alternatives should therefore be conducted in terms of a subjective assessment of how that variance will expand over the planning horizon. This section reports on the results of using a simplified version of the Nordhaus-Yohe probabilistic scenario methodology to offer a preliminary investigation of exactly this issue for a specific global emissions reduction policy—a 20\% reduction in worldwide carbon emissions from 1990 levels to be accomplished linearly by the year 2010 and maintained thereafter.\textsuperscript{17}

The Nordhaus-Yohe model, outlined in a simplified form in Appendix B, builds upon a global aggregate production function that relates world GDP at any time in the future to the employment of labor and the consumption of fossil and nonfossil fuel. The precise values of the variables that drive the model through the middle of the next century are, of course, unknown. Researchers have, at best, only subjective views of the relative likelihoods across ranges for each of these variables; they are, from our modeling perspective, simply random variables for which we can but suggest subjective probability distributions. The original Nordhaus-Yohe model included 10 random parameters, but their probabilistic Monte Carlo simulations produced a ranking of the most important sources of uncertainty in our understanding of the likely trajectory of global carbon emissions.\textsuperscript{18}

Of these original 10 variables in the Nordhaus-Yohe work, the four most important are reflected in the simplified model used here: the rate of growth of labor, the rate of growth of productivity, a price measure of technological bias toward or away from nonfossil fuel, and a depletion factor incorporated explicitly into the price of fossil fuel. The first two of these sources of uncertainty are simply rates of change that determine the future trajectory of available labor expressed in efficiency units. The third recognizes that future technological change in the energy sector need not be symmetric; ceteris

\textsuperscript{16}The structure described here and in Appendix B is a simplification of a more detailed model presented in Nordhaus & Yohe (18).

\textsuperscript{17}The notion is that emissions will be reduced by 5\% of 1990 levels over each of the four five-year intervals between 1990 and 2010; they will then be maintained at 80\% of 1990 levels every year thereafter. See Gaskins & Weyant (7) for a more complete description of this scenario and its attraction as a modeling exercise.

\textsuperscript{18}See Table 2.1 in Nordhaus & Yohe (14) for a complete ordering. The text surrounding that table describes the set of 10 variables more completely.
paribus, the price of fossil fuel may rise or fall relative to the price of nonfossil fuel. Finally, the depletion factor reflects the sensitivity of future prices of fossil fuel to remaining reserves; scarcity rents and the increasing cost of extracting fuel both work through this factor to give the price of fossil fuel an upward trajectory over time. Sampling over the values that these four variables might reasonably assume can therefore be expected to provide a reasonable portrait of the uncertainty with which we can view the trajectory of future carbon emissions, and therefore the importance of correctly designing a control strategy.

Table 1 records the baseline trajectory of emissions derived by setting all of these variables equal to their middle values (see Table 5 in Appendix B). It conforms well with the EMF-12 results, lying among the higher uncontrolled paths reported there. Table 1 records, as well, the carbon tax required to bring emissions along this baseline down to 80% of 1990 levels by the year 2010 and hold them there through the year 2050. This tax series also conforms to the consensus reported by EMF-12. We can be confident, therefore, that the probabilistic simulations based upon other possible values for the four driving variables straddle a commonly held “best guess” of what the future might hold.

Table 2 and Figure 1 both provide a broader insight into the range of possible futures. Both display the interquartile range and the 5% extremes for

<table>
<thead>
<tr>
<th>Year</th>
<th>Emissions</th>
<th>Tax</th>
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<tbody>
<tr>
<td>1990</td>
<td>100</td>
<td>25</td>
</tr>
<tr>
<td>1995</td>
<td>118</td>
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<td>458</td>
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<tr>
<td>2050</td>
<td>242</td>
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</table>

*a Indexed so that 1990 emissions equal 100.

*b Denominated in 1990 dollars per metric ton of carbon content.

19See Gaskins & Weyant (7) for a complete description of uncontrolled trajectories from 14 different energy models.
uncontrolled emissions that emerge when the other possible values of the critical driving variables are sampled across their subjective probability distributions. It is interesting to note that the median, 50th-percentile trajectory generated by this sampling lies slightly above the baseline in the early years, but then falls below the baseline in the more distant future.

Table 3 and Figure 2 turn to the heart of the matter suggested in Section 1. Percentile trajectories for emissions given the baseline tax trajectory of Table 1 are recorded there, providing graphic representation of the variance in emissions that can be expected from our current perspective given a specific carbon tax future computed for the baseline trajectory. There are possible

![Figure 1] Trajectories of unconstrained carbon emissions (indexed so that 1990 levels = 100).

<table>
<thead>
<tr>
<th>Year</th>
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<th>50th</th>
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<td>142</td>
<td>188</td>
<td>228</td>
<td>277</td>
<td>366</td>
</tr>
<tr>
<td>2045</td>
<td>142</td>
<td>190</td>
<td>233</td>
<td>285</td>
<td>382</td>
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<tr>
<td>2050</td>
<td>141</td>
<td>191</td>
<td>237</td>
<td>293</td>
<td>397</td>
</tr>
</tbody>
</table>

*Indexed so that 1990 emissions equal 100.
futures which cannot be discounted for which the baseline tax is too severe—reducing emissions far below the targeted 80%. There are, as well, trajectories for which the baseline is too lenient with emissions actually increasing from 1990 levels despite the tax. A range that encompasses 50% of the range of possible futures, in fact, shows emissions by 2050 reflecting as much as a 40% reduction (along a path characterized by relatively low population growth, low productivity growth, strong price sensitivity to resource stock depletion, and/or strong technological bias toward nonfossil

![Graph showing carbon emission trajectories given the baseline tax, which achieves a 20% reduction by the year 2010 along the median scenario (indexed so that 1990 emissions = 100).]

**Table 3** Carbon emissions* given the baseline tax achieving 20% reduction by the year 2010

<table>
<thead>
<tr>
<th>Year</th>
<th>5th</th>
<th>25th</th>
<th>50th</th>
<th>75th</th>
<th>95th</th>
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</thead>
<tbody>
<tr>
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<td>100</td>
<td>100</td>
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<td>2010</td>
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<td>58</td>
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<td>81</td>
<td>93</td>
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<td>2020</td>
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<td>2035</td>
<td>47</td>
<td>64</td>
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<tr>
<td>2040</td>
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<tr>
<td>2050</td>
<td>39</td>
<td>60</td>
<td>79</td>
<td>106</td>
<td>161</td>
</tr>
</tbody>
</table>

*Indexed so that 1990 emissions equal 100.
fuel) and a 6% increase (along a path characterized by the opposite conditions).

Table 4 and Figure 3 finally look at the orthogonal problem—the range of taxes required to guarantee the desired 20% reduction. Recall that this range also reflects the range of prices for permits that would guarantee compliance. The 50% innerquartile range recorded there shows, for example, taxes (constraint shadow prices) running all the way from 67% of the expected $435 on the low side to 148% of the same expectation on the high side. It

![Figure 3](image)

**Figure 3** Carbon taxes required to sustain a 20% reduction in carbon emissions through and beyond the year 2010 over the range of possible scenarios (denominated in 1990 dollars per ton of carbon).

**Table 4** Taxes required to achieve and sustain the 20% reduction through and beyond the year 2010

<table>
<thead>
<tr>
<th>Year</th>
<th>5th</th>
<th>25th</th>
<th>50th</th>
<th>75th</th>
<th>95th</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
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<td>22</td>
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</tr>
<tr>
<td>2015</td>
<td>123</td>
<td>181</td>
<td>238</td>
<td>312</td>
<td>460</td>
</tr>
<tr>
<td>2020</td>
<td>132</td>
<td>202</td>
<td>272</td>
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<td>141</td>
<td>223</td>
<td>308</td>
<td>424</td>
<td>673</td>
</tr>
<tr>
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<td>149</td>
<td>244</td>
<td>345</td>
<td>488</td>
<td>802</td>
</tr>
<tr>
<td>2035</td>
<td>158</td>
<td>269</td>
<td>390</td>
<td>564</td>
<td>959</td>
</tr>
<tr>
<td>2040</td>
<td>167</td>
<td>294</td>
<td>435</td>
<td>644</td>
<td>1131</td>
</tr>
</tbody>
</table>
must be emphasized, once again, that the tax considered here could be changed over time as the future unfolds and people come to understand that their policy is too restrictive or too lenient. The likelihood that these changes will occur does nothing to the importance of emissions variation given a tax in determining whether or not a marketable permit scheme with certainty equivalent emissions targets would be a better control design. The significance of design choices will persist, and the correct choice will qualitatively depend upon the same parameters.

3. CONCLUDING REMARKS AND PERSPECTIVE

The analytics of Sections 1 and 2 may appear technical, but they are commonly misunderstood even though they are both based upon fundamental concepts of economic theory. The basic point of Section 1 is, for example, that any regulatory scheme designed to be applied to situations in which individuals, governments, institutions, and other economic actors will respond to unforeseen and unforeseeable events will be misspecified, ex post; and so it will necessarily produce a welfare loss in the state of the world that actually appears. Applied to the issue of restricting carbon emissions, the notion is that restricting total emissions to a level that is too low must correspond to suboptimally low output, and so the appropriate measure of the associated welfare loss is the sum of the net values of each unit foregone—the foregone marginal benefit of the output that is not produced (because fossil fuel consumption is too low) minus the marginal social cost of the extra carbon emissions that would have resulted had the correct level of consumption been allowed. Suboptimally high emissions similarly correspond to suboptimally excessive output, and the appropriate measure of the associated welfare loss in this opposite circumstance is the sum of the net cost of the excess—the marginal social cost of emissions associated with the extra output minus the marginal benefit derived from exaggerated consumption. The resulting dead-weight losses can be catalogued case by case and aggregated into an estimate of the expected value associated with a particular policy design—the only statistic reasonably available to a regulatory authority that does not know which future state of nature will actually occur. Its choice of policy design can thus be characterized as one of choosing the alternative with the smallest net welfare loss.

Two fundamental points underlie Section 2. First of all, the demand for fossil fuel is a derived demand. It exists only because fossil fuel can be used to produce something of value. Its demand must therefore be derived directly from a production relationship so that its representation is consistent with complementary demands for other factors of production. Secondly, the translation of demand relationships into quantities actually demanded in any
particular circumstance is defined by the mechanics of the interaction of supply and demand—the building blocks of all economics. The uncertainty with which researchers view the future of carbon emissions and global change can therefore be investigated, at least in part, in terms of the uncertainty that surrounds our understanding of the parameters that will drive demand (technological change, labor productivity, population) and supply (technological change, resource availability, and resource depletion).

Having accepted these fundamentals, the results recorded at the end of Section 2 portray enormous variability in the range of carbon emissions that can be supported by reasonable specifications of future supply and demand structures for fossil and nonfossil fuels. Assuming that an agreement to limit carbon emissions has been made, the results of Section 1 therefore suggest (a) that the choice of regulatory strategies will be an important one, and (b) that the choice between a tax mechanism and a marketable permit scheme will turn on a comparison of marginal benefit and social cost parameters.

In a developed country where many major sectors would not be affected at all by greenhouse warming and where adaptation to whatever effects were felt would greatly ameliorate their potential cost, for example, a case can be made that the marginal social cost curve for carbon emissions should be relatively flat. Nordhaus (19) casts analyses provided by the US Environmental Protection Agency in its 1988 and 1989 reports to Congress against the national income accounts of the United States to produce a moderate estimate of total damage associated with an effective doubling of atmospheric carbon concentrations by the year 2050. The report of the Adaptation Panel of the National Academy of Sciences of the United States (20) underscores this modest vulnerability, especially given the relatively rapid turnover of private capital and social infrastructure that marks the US economy. The MINK Study conducted by Resources for the Future easily portrays adaptive strategies with existing technology that insulate the profitability of the agricultural sector from a hypothetical return of the 1930s dust bowl climate to the midsection of the United States.20 Informed by these and other results, Schelling (23) articulates the conclusions of most western economists who have looked at prospective damages when he concludes that “in the United States and probably Japan, Western Europe and other developed countries, the impact on economic output (of global warming) will be negligible and unlikely to be noticed” (page 8). If these conclusions hold true, of course, then the comparative advantage statistic computed in Section 1 would support a strong preference for adopting the tax alternative. Efficient reaction to changes on

20 See Crosson & Katz (21) and Easterling, McKenney, Rosenberg, and Lemon (22) for details of Resources for the Future’s work on agriculture.
the demand side of consumption would be maximized under a tax, and the associated increase in the expected social cost of emissions would be quite small.

In a developing country where a larger proportion of economic activity might be climate sensitive and where the potential for adaptation would be small, however, the marginal social cost curve for emissions could be quite steep. The Impacts Assessment of the Intergovernmental Panel on Climate Change (24) certainly catalogs a wide range of physical effects, but few of their economic ramifications over the course of the next 50 years or so have been investigated thus far. There is, in fact, a paucity of careful analysis of the economic impact of global warming on developing countries. Those that exist do, however, tend to support the suggestion that marginal costs associated with global warming are sharply rising. Broadus (25), for example, shows an extreme vulnerability of agricultural output in Egypt and Bangladesh to even a modest amount of sea level rise. Even Schelling (23) hedges somewhat by suggesting a “Marshall Plan” approach to assisting developing countries in their emissions abatement programs. It is not a large leap from there to include assistance in adaptation strategies that would reduce their vulnerability to the impacts. Absent such a program, though, the comparative advantage statistic would likely turn to favor the marketable permit alternative regardless of how they were initially distributed. The developing world would simply like to avoid the added cost associated with the added uncertainty of exactly how the developed world would respond to the abatement policy environment.

Perhaps the most significant insight of fighting through the analytics can now be articulated. International negotiations intended to craft a global response to the greenhouse warming potential of carbon emissions will have to confront and overcome divergent opinions on many issues. They will certainly start with questions of how far to reduce emissions; some countries will advocate large reductions in emissions, while others will be much more conservative. They will certainly continue to discuss how the burdens of achieving the agreed-to target might be distributed and where to allocate any revenue that regulation might collect; some countries will argue that developed countries should bear the burden of emissions reductions because they created the problem, while others will argue that the entire world would be better off if the burden were shared more equally so that the developed world can continue to grow quickly and thereby act as the catalyst for progress across the rest of the world. The results reported here suggest, however, that the debate will not be over until the negotiators have overcome equally contentious positions about what regulatory mechanism to use; the developed world might argue strongly for a tax mechanism even if the revenue does not stay at home, while the developing world might argue for an equivalent marketable permit structure regardless of how they are distributed.
APPENDIX A

Figure 4 provides some geometry that makes the analytics of this exploration most tractable.\(^{21}\) The original individual derived demand curves for the two firms are drawn in Panels A and B, but they now represent expected demand schedules from the perspective of the regulatory authority. They sum horizontally to the expected aggregate derived demand curve [also expected marginal benefit denoted \(EMB\)] shown in Panel C. An expected marginal social cost curve [denoted \(MC\)] is also displayed in Panel C. Its intersection with \(EMB\) identifies the total quantity \(F^{**}\) that maximizes expected social welfare. The same intersection also specifies the certainty equivalent price \(P^{**}\), and thus the alternative tax \(t^*\) from equation 4 in the body of the paper.

Figure 4 also displays a possible alternative state of nature in which both \(x_1\) and \(x_2\) are positive. The actual aggregate derived demand curve (actual marginal benefit) is represented in Panel C by \(MB(F_1; F_2; x_1, x_2)\); its component parts are shown in Panels A and B as \(MB_1(F; F_1; x_1)\) and \(MB_2(F; F_2; x_2)\), respectively. The socially optimal allocation of fossil fuel, \(F_1^*(x_1)\) and \(F_2^*(x_2)\), can now be described as solutions to

\[
MB_1(F_1^*(x_1); x_1) = MC[F_1^*(x_1) + F_2^*(x_2)] \quad \text{A.1}
\]

\[
MB_2(F_2^*(x_2); x_2) = MC[F_1^*(x_1) + F_2^*(x_2)] \quad \text{A.2}
\]

Equations A.1 and A.2 combine, in light of the linearity of the individual firms’ derived demand curves, to show that

\[
F_1^*(x_1) = [(x_1 + x_2)/b] F_2^*(x_2). \quad \text{A.3}
\]

Manipulating equations A.1 and A.2 in light of equation A.3 then shows that

\[
F_1^*(x_1) = [(a + x_1 - c_0)/(2c_1 + b)] - [c_1(x_1 - x_2)/b(2c_1 + b)];
\]

\[
F_2^*(x_2) = [(a + x_2 - c_0)/(2c_1 + b)] - [c_1(x_2 - x_1)/b(2c_1 + b)]; \quad \text{and}
\]

\[
F_1^*(x_1) + F_2^*(x_2) = [2(a - c_0)/(2c_1 + b)] + [(x_1 + x_2)/(2c_1 + b)]
\]

\[= F^* + (x_1 + x_2)/(2c_1 + b) \quad \text{A.4}\]

These quantities are identified in the three panels of Figure 4 along their respective derived demand curves by points \(E_1^*, E_2^*,\) and \(E^*\), respectively.

\(^{21}\) Specific, illustrative numbers were used to construct Figure 4; the reader is welcome to use them to work through the arithmetic of the modeling presented here and exploited in the text. (Subscripts and superscripts are indicated in the text but not the figure.)
Figure 4 A. Representative demand (marginal benefit) curves for fossil fuel for Firm 1. B. Representative demand (marginal benefit) curves for fossil fuel for Firm 2. C. Representative market demand (marginal benefit) curves for fossil fuel given the two firms of Panels A and B cast against marginal social cost.
The optimal tax would, meanwhile, elicit reactions $F^T_1(x_1)$ and $F^T_2(x_2)$ characterized by

$$MB_1(F^T_1(x_1); x_1) = MB_2(F^T_2(x_2); x_2) = c_0 + t^*.$$ 

Simple algebra again works through the linear structure to show that these reactions are suboptimal for the given $X = (x_1, x_2)$:

$$F^T_1(x_1) = F^*_1 + [x_1/b] > F^*_1;$$

$$F^T_2(x_2) = F^*_2 + [x_2/b] > F^*_2;$$

and

$$F^T(X) = F^T_1(x_1) + F^T_2(x_2) = F^* + (x_1 + x_2)/b > F^*.$$ 

These points are reflected in Figure 4 by points ET1, ET2, and ET, respectively. Notice in Panel C that the triangle defined by points E*, ET, and H represents the welfare loss associated with the suboptimal tax response—excess social cost of higher than optimal emissions that are not covered by the expanded consumer surplus of higher than optimal consumption. Geometry shows that the area of that triangle, the dead weight loss associated with the tax option [denoted $DWL(t^*)$], can be written as

$$DWL(t^*) = [1/4][4(c_1)/(2c_1 + b)][(x_1 + x_2)/b]^2.$$ 

Recalling that the $x_i$ have zero means, the expected value of all such losses over all combinations of the $x_i$ can be expressed simply as

$$E\{DWL(t^*)\} = [1/4][4(c_1)/(2c_1 + b)]2\{Var[F^T(X)]\},$$

A.5

where $\{Var[F^T(X)]\}$ represents the variance in consumption allowed under the tax regime under $X$.

A scheme of marketable permits would, meanwhile, hold total consumption fixed at $F^*$. The shadow price of fossil fuel would then climb by $t(X)$ to $[c_0 + t^* + t(X)]$. The geometry of Figure 4 can be used to show that

$$t(X) = (x_1 + x_2)/2;$$

this is distance GI in Panel C. Figure 4 shows how this higher shadow price translates into consumption for the individual firms, but it is the area of the triangle described by points E*, G, and I that defines the deadweight loss associated with marketable permits—welfare loss associated with suboptimally low consumption denoted $DWL(\text{per})$. The appropriate area is

$$DWL(\text{per}) = [1/4][b_2/(2c_1 + b)][(x_1 + x_2)/b]^2.$$
The expected value of these losses over possible combinations can be similarly represented as

\[ E\{DWL(\text{per})\} = \frac{1}{4}\frac{b^2}{(2c_1 + b)}\{\text{Var}[F^T(X)]\}, \quad A.6 \]

eXpressed again in terms of the variability of consumption under the tax regime.

An evaluation of the relative merits of choosing the tax regime in lieu of the marketable permits regime can finally be conducted in terms of the expected dead weight losses recorded in equations A.4 and A.5. The sign of their difference, the comparative advantage of the tax scheme over the permit scheme [denoted \( \Delta \)], can easily indicate which would be the better choice; and the magnitude of their difference provides a measure of the importance of choosing correctly. Notice, first of all, that

\[ \Delta = E\{DWL(\text{per})\} - E\{DWL(\text{t})\} = \frac{1}{4}\frac{(b - 2c_1)^2}{(2c_1 + b)}\{\text{Var}[F^T(X)]\}, \quad A.7 \]

The sign of the comparative advantage therefore depends upon the sign of numerator \( b - 2c_1 \) or, perhaps more instructively, the sign of \( \{(b/2) - c_1\} \). The latter is the difference in the magnitudes of the slopes of the aggregate marginal benefit schedule and the marginal social cost schedule.

**APPENDIX B**

The Nordhaus-Yohe model builds upon a global aggregate production function that relates world GDP at any time \( t \) [denoted \( GDP(t) \)] to the employment of labor [denoted \( L(t) \)] and the consumption of fossil and nonfossil fuel [denoted \( F(t) \) and \( E(t) \), respectively]. A simple Cobb-Douglas structure is presumed for this exercise, so

\[ GDP(t) = A(t)L(t)^{s_l}F(t)^{s_c}E(t)^{s_m} \quad B.1 \]

where

\[ A(t) = A_0(1 + g_\lambda + g_\delta)^t \quad B.2 \]

reflects both the rate of growth of labor productivity [denoted \( g_\lambda \)] and the rate of energy-saving technological change [denoted \( g_\delta \)]. Labor is assumed to grow over time, as well, at a rate given by \( g_\lambda \), so

\[ L(t) = L_0(1 + g_\lambda)^t \quad B.3 \]
Initial conditions in both labor and technology are given by $L_0$ and $A_0$, respectively; and the parameters $s_l$, $s_c$, and $s_n$ in equation B.1 reflect the respective shares of GDP devoted to paying for labor, fossil fuel, and nonfossil fuel. The Cobb-Douglas structure assumes that these shares remain constant over time, but the inclusion of $g_s$ into the definition of $A(t)$ allows for nonprice-induced energy conservation (autonomous energy-efficiency improvement in the Manne-Richels parlance\textsuperscript{22}).

Selecting labor as the numeraire, the two critical prices driving cost-minimizing employment decisions both relate to energy. Let the price of nonfossil fuel be given by

$$P_n(t) = P_{n0}(1 + g_n)^t,$$

where $g_n$ represents a technological bias in the supply of nonfossil fuel; it is positive if technological development over time favors fossil fuel and negative if development favors nonfossil sources. The time trajectory of the price of fossil fuel is more complicated, depending upon the rate of depletion of the fossil fuel resource stock $F$ and the degree to which that depletion is reflected in the price. The Nordhaus-Yohe structure is maintained here, so

$$P_c(t) = P_{co} + T + g_1 \frac{\sum F(t)}{\sum F(t)} F(t),$$

where $g_1$ is a price-depletion factor that calibrates the sensitivity of the price of fossil fuel to the rate of depletion of fossil fuel reserves, $T$ is an exogenously imposed (carbon) tax, and $\sum F(t)$ is cumulative fossil fuel consumption from time zero through time $t$. Again, initial prices are given by $P_{co}$ and $P_{no}$ for fossil and nonfossil fuel.

Cost-minimizing conditions require that the ratio of the marginal products of the two types of energy be set equal to the ratio of their respective prices. For the Cobb-Douglas technology given in equation B.1, this condition requires simply that

$$E(t) = \left[ s_n P_c(t) / s_c P_n(t) \right] F(t),$$

so that

$$F(t) = \left\{ A(t)L(t)^{s_l} \left[ s_c / P_c(t) \right] \left[ s_n / P_n(t) \right] \right\}^{s_m(1/s_l)}$$

fully characterizes the consumption of fossil fuel over time. Since emissions [denoted $C(t)$] depend upon the consumption of fossil fuel, the link to an emissions target can be given by

\textsuperscript{22}Manne & Richels (26) use autonomous energy-efficiency improvement to quantify nonprice-induced conservation of energy, which naturally occurs as technological development moves forward.
where \( C_0 \) represents an initial ratio of carbon emissions to fossil fuel consumption and \( g_c \) represents a rate of change in that ratio over time.

Table 5 records three values for each of these parameters. Review of the published literature available in 1983 suggested that subjective relative likelihoods of 25%, 50%, and 25% could be associated with the low, middle, and high values of each. Table 5 also records the shares of GDP devoted to labor, fossil fuel, and nonfossil fuel required to specify the production technology of equation \( B.1 \). The other initial conditions are omitted because the EMF-12 convention of reporting results in terms of an index that sets 1990 values equal to 100 has been adopted. Simulation based upon equations

\[
C(t) = C_0(1 + g_c)tF(t), \quad B.7
\]

**Table 5 Parameterization of the simplified Nordhaus-Yohe model**

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Population growth(^a)</td>
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<td>High</td>
<td>0.25</td>
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<td>1.6</td>
</tr>
<tr>
<td>Middle</td>
<td>0.50</td>
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</tr>
<tr>
<td>Low</td>
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<td>1.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Labor productivity(^b)</td>
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<td>2.3</td>
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<tr>
<td>Middle</td>
<td>0.50</td>
<td>2.3</td>
<td>1.6</td>
</tr>
<tr>
<td>Low</td>
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<td>Energy price bias(^c)</td>
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<td>Middle</td>
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<td>0.0</td>
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<tr>
<td>Low</td>
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<tr>
<td>Depletion factor(^d)</td>
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<tr>
<td>Middle</td>
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</tr>
<tr>
<td>Low</td>
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<td>72</td>
<td>72</td>
</tr>
</tbody>
</table>

\(^a\)The \( g_L \) parameter recorded as percent growth in global population per year; see equation \( B.3 \).

\(^b\)The \( g_4 \) parameter recorded in percent growth in the GDP/population ratio per year; see equation \( B.2 \).

\(^c\)The \( g_n \) parameter recorded in terms of an annual percent change in the price of fossil fuel relative to the price of nonfossil fuel; see equation \( B.4 \).

\(^d\)The \( g_p \) parameter in the supply equation for fossil fuel associated in Nordhaus-Yohe (18) with a fossil fuel resource stock of \( 11 \times 10^{12} \) metric tons of coal equivalent; see equation \( B.5 \).

\(^e\)The \( g_s \) parameter (the Manne-Richels (26) autonomous energy-efficiency improvement parameter) is taken to be 0.7% per year; variability here is built into variability in \( g_s \), since \( g_s \) and \( g_4 \) enter equation \( B.2 \) additively.

\(^22\)These values are taken from Table 2.14 in Nordhaus & Yohe (14). The parameter \( g_s \) takes a fixed value equal to -0.7% per year to reflect the 30% decline in the world energy-GDP ratio that is emerging as the EMF-12 consensus; see Weyant & Sit (15) and Gaskins & Weyant (7).
B.1 through B.7 given these relative likelihood values was undertaken to explore the potential for achieving and maintaining a 20% reduction in carbon emissions by the year 2010. Results are reported in the text.

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