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Substitution and the Control of Pollution

A Comparison of Effluent Charges and Quantity Standards under Uncertainty[1]

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Single-valued price and quantity controls of a polluting activity are compared under uncertainty. The ability to substitute other inputs for the pollutant in the production of a positively valued final good, and the usual discrepancy between the amount of pollution actually consumed and the amount emitted are carefully incorporated. The first is found to influence the degree to which cost fluctuation is reflected in the output of the final good. The second concern alters the region of the benefit function into which output is inserted. Both change the welfare losses associated with random fluctuation in the costs of reducing pollution.

INTRODUCTION

The relative abilities of price and quantity controls to handle an economic activity in an uncertain environment have attracted considerable analytic attention in the recent literature (see, for example, [1, 4, 6]). When these more general studies are cast in terms of a pollution control problem, however, they fall short on two fronts. First of all, unless the pollutant and the positive product appear in fixed proportions, representing both by a single variable (as in [3]) is inappropriate. Such a procedure misses the potential substitution of output for proportionately less pollution that can be affected by the reassignment of capital. In addition, the quantity of a pollutant actually being consumed is typically related to the quantity produced by a plethora of weather related random variables. This second observation uncovers a source of significant uncertainty that has been thus far ignored. The purpose of this paper, then, is to trace the impact of these two caveats on the comparison of prices (effluent charges) and quantities (standards) in pollution control.

The first section records the certainty genesis of the one-firm model within which we will be working throughout most of the paper. Care is taken to guarantee that our two alternative controls generate precisely the same outcome in an environment of complete information. As we subsequently turn to an uncertain world, we can be sure of comparing certainty equivalent regulations; any dis-

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crepancies in outcome that we may observe are then necessarily produced by an asymmetric treatment of the random effects we will have imposed.

Three basic sources of uncertainty are set forth in the second section where the initial comparison is made. So that we may fully concentrate our attention on the relative merits of prices and quantities, we follow Weitzman's lead and compare single-valued, once and for all control specifications. It would seem, given the often espoused bureaucratic preference for simple regulations, that this limitation is not only analytically prudent, but is also asking the correct question vis à vis the special case of pollution control. Criticism of dealing with but a single firm is then answered by treating the multifirm example in Section IV. In either case, differences in the variation of (total) output and (total) pollution created by the two controls emerge as the crucial determinants of which control is better; the curvatures of the relevant parts of the social loss function will then reflect the importance of making the correct choice.

Once the basic model is fully analyzed, the impacts of substitution (Section III) and random discrepancies in the amount of pollution consumed (Section V) are handled easily. Substitution simply changes the effective curvature of the benefit function, thereby increasing or decreasing the losses created by output variation. The consumption distortion of Section V meanwhile adds a similar effect on the curvature of the pollutant's social cost function.

I. EQUIVALENT CONTROLS UNDER CERTAINTY

Pollutants are generally modeled as secondary outputs of joint production processes. Economically, however, they behave more like inputs. Increased emission levels diminish the public welfare just as economists expect people to dislike an increase in their working hours at the expense of their leisure time. Compensation must therefore be paid by the producer to the public, and constitutes a negative output price. For this reason, we will consider pollution to be a necessary input for the production of a final good (x). We can then easily focus our attention on the substitution effects we mean to study by assuming profit maximizing, price taking behavior by the producer of x, and altering the elasticity of substitution between the pollutant and a second productive factor (K). While we envision K to be a homogeneous factor that is available at a constant per unit cost (r), it can also be thought to represent an aggregate of all inputs.[3]

The production of x can thus be summarized by $x = f[K, (z + q)]$, where z reflects the amount of pollution that leaves the plant, q reflects the amount of pollution removed from the emissions stream, and $(z + q)$ therefore reflects the total amount of pollution employed in the production of x. Benefits from x are recorded by $B(x)$; the social cost of emitted pollution, by $H(z)$; and the cost of removing pollutants from the emissions stream, by $C(q)$. We finally assume that for all x, q, and z, $B'(x)$, $H'(z)$, $C'(q)$, $H''(z)$, and $C''(q)$ are strictly positive, while $B''(x)$ is strictly negative.

[3] Without this assumption, we would simply be concerned with the elasticities of substitution between the pollutant and the other inputs on an individual basis. A *ceteris paribus* change in one elasticity would have the same, albeit dampened, effect as the one we will capture. Partial equilibrium is similarly innocuous when we confine our study to either a single firm, or an industry that is small relative to the input market.

The socially optimal triple (K', q', z') can be discovered by solving

$$\max_{K,q,z} \{B(x) - rK - H(z) - C(q)\}.$$

The first-order conditions reveal some familiar results:

$$B'\{f[K', (z' + q')]\} f_1[K', (z' + q')] = r, \tag{1a}$$

$$B'\{f[K', (z' + q')]\} f_2[K', (z' + q')] = H'(z'), \tag{1b}$$

$$B'\{f[K', (z' + q')]\} f_2[K', (z' + q')] = C'(q'). \tag{1c}$$

The marginal value product of K must equal the per unit cost of K. The marginal value product of polluting must simultaneously equal the marginal cost of the portion that is cleaned up and the marginal social harm of the portion that is not cleaned up. Combining (1b) and (1c) also reveals that the marginal cleaning cost should equal the marginal social harm of not cleaning further.

The assumed shapes of the various functions guarantee the uniqueness of (K', q', z'). We need only demonstrate that (K', q', z') can be achieved under either price or quantity regulation of emissions to be assured of comparing certainty equivalent controls in the subsequent analysis of uncertainty. The logical choice for a price specification is $\tilde{p} = H'(z')$ per unit of pollution actually emitted from the plant. The producer of x (x-firm) maximizes profits by solving

$$\max_{K,q,z} \{P_x x - rK - \tilde{p}z - C(q)\},$$

where P_x is the expected price of x. The market clears only when

$$P_x = B'\{f[K', (z' + q')]\},$$

so that his first-order conditions,

$$P_x f_1 = r,$$

$$P_x f_2 = C'(q),$$

and

$$P_x f_2 = \tilde{p},$$

duplicate (1). The price order $\tilde{p}$ therefore achieves the optimum.

The best quantity order meanwhile requires that no more than z' be emitted. If we can demonstrate that the x-firm will choose K' and q' when it is so constrained, we will have proven our desired equivalence. The x-firm is now forced to confront the problem,

$$\max_{K,q} \{P_x f[K, (z' + q)] - rK - C(q)\}.$$

The first-order conditions subsequently specify that

$$P_x f_1[K, (z' + q)] = r, \tag{2a}$$

and

$$P_x f_2[K, (z' + q)] = C'(q). \tag{2b}$$

Recalling that $P_x = B'\{f[K', (z' + q')]\}$, we see that (2) duplicate (1a) and (1c). The prescribed quantity control therefore also achieves the optimum.

II. AN INITIAL COMPARISON UNDER UNCERTAINTY

Uncertainty will initially enter our model from three distinct sources. First of all, both the benefits generated by consuming x and the social costs of consuming the pollutant are presumed to be imprecisely specified; i.e., $B = B(x, \eta)$ and $H = H(z, \xi)$, where η and ξ are random vectors that reflect not only random shocks to the entire system, but also imprecise knowledge of the functions themselves. The cost of removing pollutants from emissions streams can also be subject to random changes. To reflect these effects, we assume that while the x-firm is capable of reading them, the central regulatory agency must issue its command before they appear. These changes therefore constitute ex ante uncertainty about cleansing costs, for the center, that can be summarized by writing $C = C(q, \theta)$.

A few technicalities will simplify our task. By specifying a CES production function, we can easily monitor the impact of substitution; thus,

$$x = [\gamma K^\rho + (1 - \gamma)(z + q)^\rho]^{1/\rho}.$$

The elasticity of substitution (σ) is then simply $[1/1 - \rho]$. As ρ approaches 1, substitution is easy, reflecting switches to nonpolluting techniques or production of pollutants that are easily neutralized. When ρ becomes arbitrarily negative, however, little substitution is possible; alternative techniques that produce a treatable pollutant are unavailable. We will also be assuming that θ, ξ, and η are all independently distributed. Footnotes will record the effect of omitting this final assumption.

The *ex ante* optimum can be represented by the triple $(\hat{K}_0, \hat{z}_0, \hat{q}_0)$ that maximizes

$$E\{B(x, \eta) - rK - H(z, \xi) - C(q, \theta)\}. \tag{3}$$

If we assume that the cost and benefit functions have the correct shapes for all (θ, ξ, η) then $(\hat{K}_0, \hat{z}_0, \hat{q}_0)$ exists uniquely. The resulting production of x is, of course,

$$\hat{x}_0 = [\gamma \hat{K}_0{}^\rho + (1 - \gamma)(\hat{z}_0 + \hat{q}_0)^\rho]^{1/\rho}.$$

Our comparison becomes most tractible if we approximate each function with its second-order Taylor expansion around $\hat{q}_0$, $\hat{z}_0$, and $\hat{x}_0$. Thus

$$B(x, \eta) = b(\eta) + [B' + \beta(\eta)](x - \hat{x}_0) + \tfrac{1}{2}B_{11}(x - \hat{x}_0)^2,$$

$$C(q, \theta) = a(\theta) + [C' + \alpha(\theta)](q - \hat{q}_0) + \tfrac{1}{2}C_{11}(q - \hat{q}_0)^2,$$

and

$$H(z, \xi) = h(\xi) + [H' + \delta(\xi)](z - \hat{z}_0) + \tfrac{1}{2}H_{11}(z - \hat{z}_0)^2.$$

By defining the first-order coefficients in terms of means and disturbances (i.e., $B' = EB_1(\hat{x}_0, \eta)$ and $\beta(\eta) = B_1(\hat{x}_0, \eta) - B'$), we can observe that $E\alpha(\theta) = E\beta(\eta) = E\delta(\xi) = 0$. If the random variables are compact and their variances small, Samuelson assures us that second-order approximation costs us nothing (see [2]). Subsequent analysis of simpler control comparisons has, in addition, revealed that very little substantive economics is lost by stopping after three terms (see [5]).[4]

[4] The effects that are subsumed by this assumption are easily rationalized in a geometric context Section V will provide an example. It should also be noted that by assuming the usual shapes for B, H, and C (see Section I) for all possible θ, η, and ξ, we can guarantee that of the constants appearing in the approximation, only B_{11} can, and will, be negative.

The first-order conditions that characterize $(\hat{K}_0, \hat{z}_0, \hat{q}_0)$ as the solutions of (3) emerge from these restrictions in the form

$$\gamma B'[\hat{x}_0/\hat{K}_0]^{1-\rho} = r, \tag{4a}$$

$$(1-\gamma)B'[\hat{x}_0/(\hat{z}_0 + \hat{q}_0)]^{1-\rho} = H', \tag{4b}$$

$$(1-\gamma)B'[\hat{x}_0/(\hat{z}_0 + \hat{q}_0)]^{1-\rho} = C'. \tag{4c}$$

By combining (4b) and (4c), we see that $H' = C'$; (4a) and (4b) similarly reveal that

$$[\gamma/(1-\gamma)][(\hat{z}_0 + \hat{q}_0)/\hat{K}_0]^{1-\rho} = (r/H').$$

If we define $A \equiv [\gamma H'/(1-\gamma)r]$, we can write

$$\hat{K}_0 = A^{\sigma}(\hat{z}_0 + \hat{q}_0),$$

$$\hat{x}_0 = (\hat{z}_0 + \hat{q}_0)[\gamma A^{\sigma\rho} + (1-\gamma)]^{1/\rho} \equiv (\hat{z}_0 + \hat{q}_0)G(\rho),$$

and Eqs. (4) reduce to

$$\gamma B'(\hat{x}_0/\hat{K}_0)^{1-\rho} \equiv \gamma B'\left[\frac{[\gamma A^{\sigma\rho} + (1-\gamma)]^{1/\rho}}{A^{\sigma}}\right]^{1-\rho} = r \tag{4a$'$}$$

and

$$(1-\gamma)B'[\hat{x}_0/\hat{z}_0 + \hat{q}_0)]^{1-\rho} \equiv (1-\gamma)B'\{[\gamma A^{\sigma\rho} + (1-\gamma)]^{1/\rho}\}^{1-\rho} = H' = C'. \tag{4b$'$) and (4c$'$}$$

These notations will be used later.

1. The Optimal Price Order

For any price order (effluent tax) requiring payment on emissions, the x-firm will read θ and choose the level of pollution the level of K, and thus the output level, that maximizes profits. We have previously constrained this firm to be price taker, and will now assume that its expectations about the price of x are accurate, i.e., if $p_x(\eta)$ represents the perceived price distribution, $Ep_x(\eta) = B'$.[5] The x-firm will thus maximize

$$\{B'\{[\gamma K(t,\theta)^{\rho} + (1-\gamma)[z[t,\theta) + q(t,\theta)]^{\rho}\}^{1/\rho} - rK(t,\theta) - C[q(t,\theta),\theta] - t[z(t,\theta)]\}$$

with respect to $K(t, \theta)$, $z(t, \theta)$, and $q(t, \theta)$, for any specified t and observed θ. The first-order conditions require that

$$\gamma B'\{\{\gamma K(t,\theta)^{\rho} + (1-\gamma)[z(t,\theta) + \alpha(t,\theta)]^{\rho}\}^{1/\rho}/K(t,\theta)\}^{1-\rho} = r, \tag{5a}$$

$$(1-\gamma)B'\{[\text{—}]^{1/\rho}/[z(t,\theta) + q(t,\theta)]\}^{1-\rho} = t, \tag{5b}$$

$$(1-\gamma)B'\{[\text{—}]^{1/\rho}/[z(t,\theta) + q(t,\theta)]\}^{1-\rho} = C' + \alpha(\theta) + C_{11}[q(t,\theta) - \hat{q}_0]. \tag{5c}$$

Equations (5b) and (5c) combine to reveal that

$$q(t,\theta) = \hat{q}_0 - (C' + \alpha - t)/C_{11}.$$

Observe, however, that random changes in the marginal cost of cleaning effluent have no effect on the marginal products of either K or the pollutant. Their employment should therefore be independent of the value of θ that actually appears.

[5] Footnote 8 will record the impact of weakening this presumption.

We can formalize this notion by suggesting that $K(t, \theta) \equiv K(t)$, and

$$z(t, \theta) = z(t) + (C' + \alpha - t)/C_{11}.$$

Subsequent observations will verify these suggestions.

The center is assumed fully aware of these reaction functions and maximizes

$$E\{B\{[\gamma K(t)^{\rho} + (1 - \gamma)(z(t, \theta) + q(t, \theta))^{\rho}]^{1/\rho}, \eta\} - rK(t) \\ - H[z(t, \theta), \xi] - C[q(t, \theta), \theta]\} \quad (6)$$

with respect to t. The resulting first-order conditions reveal, after some algebra, that the best effluent charge is $\tilde{t} = H'$. In that case,

$$\tilde{q}(\theta) = \hat{q}_0 - (\alpha/C_{11}), \quad (7a)$$

$$\tilde{z}(\theta) = \hat{z}_0 + (\alpha/C_{11}), \quad (7b)$$

$$\tilde{K}(\theta) = \hat{K}_0, \quad (7c)$$

and

$$\tilde{x}(\theta) = \hat{x}_0 \quad (7d)$$

reflect the x-firm's responses to an observed θ;[6] only in the context of these reactions do the maximization conditions for (6) reduce to the accurate statement that $H' = C'$.

2. The Optimal Quantity Order

We need to determine similar reaction functions for the x-firm under an arbitrary quantity order $(\bar{z})$. In this case, the firm solves

$$\max_{K,q}\{B'[\gamma K^{\rho} + (1 - \gamma)(\bar{z} + q)^{\rho}]^{1/\rho} - rK - C(q, \theta)\}$$

to determine its reaction functions. These reactions, notationally represented by $q(\bar{z}, \theta)$ and $K(\bar{z}, \theta)$, emerge from two rather complex first-order conditions:

$$\gamma B'\{\{\gamma K(\bar{z}, \theta)^{\rho} + (1 - \gamma)[\bar{z} + q(\bar{z}, \theta)]^{\rho}\}^{1/\rho}/K(\bar{z}, \theta)\}^{1-\rho} = r, \quad (8a)$$

$$(1 - \gamma)B'\{\{\gamma K(\bar{z}, \theta)^{\rho} + (1 - \gamma)[\bar{z} + q(\bar{z}, \theta)]^{\rho}\}^{1/\rho}/[\bar{z} + q(\bar{z}, \theta)]\}^{1-\rho} \\ = C' + \alpha + C_{11}[q(\bar{z}, \theta) - \hat{q}_0]. \quad (8b)$$

The center meanwhile takes the emerging $q(\bar{z}, \theta)$ and $K(\bar{z}, \theta)$ as given, and maximizes

$$E\{B\{f[K(\bar{z}, \theta), \bar{z} + q(z, \theta)], \eta\} - rK(\bar{z}, \theta) - H(\bar{z}) - C[q(\bar{z}, \theta), \theta]\} \quad (9)$$

with respect to $\bar{z}$ to determine the best quantity order, $\hat{z}$. If we assert that

$$\hat{z} = \hat{z}_0, \quad (10a)$$

$$q(\hat{z}_0, \theta) = \hat{q}_0 - (\alpha/C_{11}), \quad (10b)$$

and

$$K(\hat{z}_0, \theta) = A^{\sigma}[\hat{z}_0 + q(\hat{z}_0, \theta)], \quad (10c)$$

[6] Equations (5) reduce to (4) given these reactions. Equations (7) therefore solve system (5), and their uniqueness is guaranteed by the assumed shapes of the various cost and benefit schedules.

Eqs. (8) reduce to (4′):

$$\gamma B' \left\{ \frac{\{(\gamma\{A^{\sigma}[\hat{z}_0 + \hat{q}_0 - (\alpha/C_{11})]\}^{\rho}) + (1-\gamma)[\hat{z}_0 + \hat{q}_0 - (\alpha/C_{11})]^{\rho}\}^{1/\rho}}{A^{\sigma}[\hat{z}_0 + \hat{q}_0 - (\alpha/C_{11})]} \right\}^{1-\rho}$$
$$= \gamma B'\{[\gamma A^{\sigma\rho} + (1-\gamma)]^{1/\rho}/A^{\sigma}\}^{1-\rho} = r;$$

$$(1-\gamma)B' \left\{ \frac{\{(\gamma\{A^{\sigma}[\hat{z}_0 + \hat{q}_0 - (\alpha/C_{11})]\}^{\rho}) + (1-\gamma)[\hat{z}_0 + \hat{q}_0 - (\alpha/C_{11})]^{\rho}\}^{1/\rho}}{[\hat{z}_0 + \hat{q}_0 - (\alpha/C_{11})]} \right\}^{1-\rho}$$
$$= (1-\gamma)B'[\gamma A^{\sigma\rho} + (1-\gamma)]^{1-\rho}$$
$$= C' + \alpha + C_{11}[\hat{q}_0 - (\alpha/C_{11}) - \hat{q}_0] = C'.$$

The first-order condition for (9) also reduces to (4c′) under these assertions; definitions guarantee the validity of (4′), and uniqueness allows us to conclude that Eqs. (10) are correct. Under the quantity regime, therefore, emissions are held constant, while output and the utilization of K vary with θ:

$$\hat{q}(\theta) = \hat{q}_0 - (\alpha/C_{11}), \tag{11a}$$

$$\hat{K}(\theta) = \hat{K}_0 - A^{\sigma}(\alpha/C_{11}), \tag{11b}$$

and

$$\hat{x}(\theta) = \hat{x}_0 - G(\rho)(\alpha/C_{11}). \tag{11c}$$

The cleansing response to random shifts in marginal costs is the same under both modes; increases (decreases) in costs cause the amount of pollution removed from the effluent to fall (rise). The similarity stops there. Under quantity standards, emissions are fixed so that total pollution must fall, but the employment of K and final output vary inversely with cleaning costs. Output and K employment are meanwhile fixed under effluent charges; it is emissions that vary directly with cleaning costs. These observations will weigh heavily on the welfare comparison of the two modes.

3. *The Comparative Advantage of Prices*

The means by which we will compare the two alternatives will be the comparative advantage of prices over quantities (Δ): the difference between the expected levels of benefits minus costs achieved by the optimal effluent charge, $\tilde{t}$, and the corresponding optimal quantity standard, $\hat{z}_0$. Mathematically,

$$\begin{aligned}\Delta \equiv E\{[B(\hat{x}_0, \eta) - r\hat{K}_0 - H([\hat{z}_0 + (\alpha/C_{11})], \xi) \\ - C([\hat{q}_0 - (\alpha/C_{11})], \theta)] - [B([\hat{x}_0 - G(\rho)(\alpha/C_{11})], \eta) \\ - r[\hat{K}_0 - A^{\sigma}(\alpha/C_{11})] - H(\hat{z}_0, \xi) - C([\hat{q}_0 - (\alpha/C_{11})], \theta)]\}.\end{aligned}$$

When Δ is positive, then, an effluent charge is preferred; when it is negative, a quantity standard is preferred.

By observing that $E\{r\alpha/C_{11}\} = E\{\beta(\eta)[\alpha(\theta)/C_{11}]\} = E\{\delta(\xi)[\alpha(\theta)/C_{11}]\} = 0$, we can derive an interpretable expression for Δ:

$$\begin{aligned}\Delta &= -\tfrac{1}{2}\{H_{11}\,\mathrm{Var}\,[\tilde{z}(\theta)] + B_{11}\,\mathrm{Var}\,[\hat{x}(\theta)]\} \\ &= -\tfrac{1}{2}\{H_{11} + B_{11}[G(\rho)]^2\}\,\mathrm{Var}\,(\alpha/C_{11}).\end{aligned} \tag{12}$$

While the first line provides the clue to understanding the underlying economics, the second is analytically simpler and is recorded for future use.[7] It is, then, the

TABLE I

The Comparative Advantage at the Curvature Extremes

Limit	Interpretation	Qualifications	Δ
$H_{11} \to 0$	Linear social costs	(none)	$(+)$
$H_{11} \to \infty$	Highly curved social costs	(none)	$-\infty$
$B_{11} \to 0$	Linear benefits	(none)	$(-)$
$B_{11} \to -\infty$	Highly curved benefits	(none)	$+\infty$
$C_{11} \to 0$	Linear cleaning costs	$H_{11} > B_{11}(G(\rho))^2$	$-\infty$
$C_{11} \to 0$	Linear cleaning costs	$H_{11} < B_{11}(G(\rho))^2$	$+\infty$
$C_{11} \to \infty$	Highly curved cleaning costs	(none)	0

variance in the argument of any part of the social welfare function that causes trouble. The term $\tilde{z}(\theta)$, for instance, indicates how actual emissions vary with θ under effluent charges; that movement creates a level of expected costs that is higher than the level that would be achieved if $\hat{z}_0$ were emitted with certainty. The magnitude of that loss is simply H_{11} Var $[\tilde{z}(\theta)]$, and, since it is a loss created by prices, must be subtracted from the comparative advantage of prices. The expression $\hat{x}(\theta)$ similarly indicates how θ effects output, thereby achieving a level of expected benefits that is lower than the level that would be generated by the certain production of $\hat{x}_0$. The magnitude of this second loss is $\{-B_{11}$ Var $[\hat{x}(\theta)]\}$, and, since it is created by quantity control, it must appear in Δ as a positive expression. The two curvature parameters (B_{11} and H_{11}) clearly reflect which variation effect is the more deleterious.[8]

Table I summarizes the extreme cases, and should reinforce the reader's understanding. The impact of C_{11}, the curvature of the cleansing cost function, may be a bit mysterious at first glance. Observe, however, that as C_{11} increases arbitrarily (nears zero), the profit maximizing responses to θ all approach zero (increases without bound). In the limits, therefore, the comparison either becomes moot (since both modes produce the same circumstance) or turns crucially on the sign of $\{H_{11} + B_{11}[G(\rho)]^2\}$.

[7] Without the assumption of independence between θ, ξ, and η, the expression

$$-\{\mathrm{Cov}\,[\beta(\eta)\,;\hat{x}(\theta)] + \mathrm{Cov}\,[\delta(\xi), \tilde{z}(\theta)]\}$$

is added to Δ. These new terms reflect the correlation effect of randomly shifting output in the context of randomly shifting marginal benefits (e.g.). If Cov $(\beta\,;\hat{x})$ is positive, for instance, output tends to increase just as marginal benefits are shifting upward. That would be, of course, the correct direction, and should create a positive influence for the control that allows it—quantities. That effect is accurately subtracted from Δ, the comparative advantage of prices.

[8] Were the firm to operate with inaccurate information about the price of x, it could easily be computing a price mean different from the true mean B'. The center could easily adjust its control orders to compensate completely for this error if it knew of the problem, and the same distributions of output and pollution could be achieved. Nothing would have changed. If, on the other hand, the center were unaware of this informational difficulty, output and pollution would emerge from around the wrong means. These distortions in the average outcomes would, of course, produce dead weight losses under either control in addition to the variation spawned losses we have already observed. By properly manipulating the resulting comparative advantage of prices, however, it is possible to summarize both effects with the variance of output and pollution around the incorrect means computed by the firm. The interpretations that we have attached to Eq. (11), and thus the importance of variation in output and the level of pollution, therefore survive intact when we simply remember that variation must be measured around the means computed by the firm. It follows that the substantive conclusions are unchanged.

III. THE IMPACT OF SUBSTITUTIBILITY

We have already noted that the ability to substitute K for q in the face of changes in cleansing costs is utilized under quantity standards alone. We are now in a position to explain why. Effluent charges are relevant only on the margin. Any changes in costs that might then occur under a price control result only in changes in the proportion of the fixed amount of total pollution that is emitted. No substitution takes place and output is unaltered. Equations (10) reveal, however, that substitution does take place on the quantity side of the comparison; it can be thought simply to reflect the degree with which cost changes influence the final production of the good x.

We need therefore only investigate the effect on $G(\rho)$ of changing ρ to trace the impact of the elasticity on the quantity side of Δ:

$$[\partial G(\rho)/\partial \rho] = -\{\gamma[\rho(1-\rho)]^{-2}G(\rho)A^{\sigma\rho}\}[\log(\gamma A^{\sigma\rho} + 1 - \gamma)]\log A.$$

The sign of this partial depends crucially on the signs of the logarithmic terms; all of the others are positive. Table II summarizes the various cases.

We can infer directly from the table that when $0 < \rho < 1$, an increase in the elasticity of substitution ($\rho \to 1$) will cause a decrease in $G(\rho)$, and thus a decrease in the severity with which cost variations are manifest in output variations under quantity control. Indeed, if K and the pollutant are perfect substitutes ($\rho = 1$), the reduction in total pollution caused by an increase (e.g.) in cleans'ng costs would be accompanied necessarily by an increase in K that would leave total output unchanged. The benefit side would therefore disappear, and a quantity standard would be unambiguously favored [i.e., $\Delta(\rho = 1) = -\frac{1}{2}H_{11}\,\mathrm{Var}\,(\alpha/C_{11}) < 0$]. The opposite conclusion is drawn when $0 < \sigma < 1$ ($\rho < 0$). The output effects under standards of cost disturbances increase from a factor of $[(1-\gamma)/C_{11}]$, when K and the pollutant are employed in fixed proportions ($\sigma = 0$), to a maximum in the Cobb–Douglas case ($\rho = 0$). The benefit side therefore increases in importance and creates a positive bias for using effluent charges.

IV. THE MULTIFIRM CASE

We have thus far analyzed only the restrictive case of a single polluting producer of the final good. An extension of the control comparison to the multifirm case will allow us to demonstrate not only the impact of increasing the number of firms, but also the robustness of our previous conclusions.

In this section, therefore, we postulate n producers of x, all of which also discharge the pollutant in question. For the ith firm, then,

$$x_i = [\gamma_i K_i^{\rho(i)} + (1-\gamma_i)(z_i + q_i)^{\rho(i)}]^{1/\rho(i)} \tag{13}$$

represents the production function, and

$$C_i(q_i, \theta_i) = a_i(\theta_i) + [C_i' + \alpha_i(\theta_i)](q_i - \hat{q}_{0i}) + \tfrac{1}{2}C_{11}^{i}(q_i - \hat{q}_{0i})^2$$

approximates the cost function for removing q_i from the effluent.[9] The optimal effluent charge emerges from these complications intact; $\tilde{t} = H'$.

[9] The central points for these approximations are determined by maximizing

$$E\{B[(\textstyle\sum x_i), \eta] - r(\sum K_i) - \sum C_i(q_i, \theta_i) - H[\sum z_i), \xi]\}$$

with respect to K_i, q_i, and z_i ($i = 1, \ldots, n$), and in the context of (13). The resulting optimal ($\{\hat{q}_{0i}\}$, $\{\hat{K}_{0i}\}$, and $\{\hat{z}_{0i}\}$) also allows us to construct second-order approximations for B and H around ($\sum \hat{x}_{0i}$) and ($\sum \hat{z}_{0i}$), respectively.

TABLE II

The Sign of $[\partial G(\rho)/\partial \rho]$

Term	$A > 1$ $0 < \rho < 1$	$A > 1$ $\rho < 0$	$A < 1$ $0 < \rho < 1$	$A < 1$ $\rho < 0$
$\log(\gamma A^{\sigma\rho} + 1 - \gamma)$	(+)	(−)	(−)	(+)
$\log A$	(+)	(+)	(−)	(−)
$(\partial G(\rho)/\partial \rho)$	(−)	(+)	(−)	(+)

The ith firm then responds as follows:

$$\tilde{q}_i(\theta_i) = \hat{q}_{0i} - [\alpha_i(\theta_i)]/C_{11}{}^i,$$
$$\tilde{z}_i(\theta_i) = \hat{z}_{0i} + [\alpha_i(\theta_i)]/C_{11}{}^i,$$
$$\tilde{K}_i(\theta_i) = \hat{K}_{0i},$$

and

$$\tilde{x}_i(\theta_i) = \hat{x}_{0i}.$$

The optimal quantity standard is, meanwhile, $\hat{z}_i = \hat{z}_{0i}$, so that

$$\hat{q}_i(\theta_i) = \hat{q}_{0i} - [\alpha_i(\theta_i)]/C_{11}{}^i,$$
$$\hat{K}_i(\theta_i) = \hat{K}_{0i} - A_i{}^{\sigma(i)}(\alpha_i/C_{11}{}^i),$$
$$\hat{X}_i(\theta_i) = \hat{x}_{0i} - G_i(\rho_i)(\alpha_i/C_{11}{}^i),$$

for all i.[10] The resulting comparative advantage of a uniform effluent charge $\tilde{t}$ over quantity standards $\{\hat{z}_{0i}\}$ for the entire "industry" should not be surprising:[11]

$$\Delta(n) = -\tfrac{1}{2}[H_{11} \operatorname{Var}(\sum_{i=1}^{n} \tilde{z}_i(\theta_i)) + B_{11} \operatorname{Var}(\sum_{i=1}^{n} \hat{x}_i(\theta_i))]$$

$$= -\tfrac{1}{2}\{H_{11} \operatorname{Var}(\sum_{i=1}^{n} [\alpha_i(\theta_i)/C_{11}{}^i)] + B_{11} \operatorname{Var}[\sum_{i=1}^{n} (G_i(\rho_i)\alpha_i(\theta_i)/C_{11}{}^i)]\}. \quad (14)$$

The intuitive interpretation of (12) that unraveled the single-firm case can be applied directly to (14), when it is cast in terms of variances of *total* industry output and *total* industry emissions. All of the previous conclusions are, therefore, equally applicable. It is, indeed, total output and total emissions that enter the benefit and social cost functions; their variances should thus create the losses upon which the control comparison turns.

The *ceteris paribus* effect on the comparative advantage of increasing the number of firms producing x, and emitting the pollutant, can also be deduced. So that we may concentrate our full attention on this effect, we now assume that all of the

[10] The constants are defined in a manner analogous to those which appeared in the single-firm analysis,

$$A_i{}^{\sigma(i)} \equiv [\gamma_i H'/(1 - \gamma_i)r]^{\sigma(i)},$$
$$\sigma(i) \equiv (1/1 - \rho_i),$$

and

$$G_i(\rho_i) \equiv [\gamma_i A_i{}^{\sigma(i)\rho_i} + (1 - \gamma_i)]^{1/\rho_i},$$

for the ith firm.

[11] For a more complete derivation of a similar problem, including the existence of $\{q_{0i}\}$ and computation of $\Delta(n)$, consult Yohe [5].

firms are identical, and that the θ_i are identically distributed; i.e.,

$$x_i = [\gamma K^\rho + (1 - \gamma)(z + q)^\rho]^{1/\rho},$$
$$c(q_i, \theta_i) = a(\theta_i) + [C' + \alpha(\theta_i)](q_i - \hat{q}_0) + \tfrac{1}{2}C_{11}(q_i - \hat{q}_0)^2,$$
$$\operatorname{Var}[\alpha(\theta_i)] = s\,,$$

and

$$\operatorname{Cov}[\alpha(\theta_i); \alpha(\theta_j)] = ks^2.$$

We must also correct for the secondary influence of n. To that end, we define a transformed cost function for all firms,

$$\Gamma(Q_i, \theta_i) \equiv nC[(Q_i/n), \theta_i],$$

that can be interpreted as total removal cost given as a function of the total amount of pollution produced but not emitted, and under the assumption that all firms are identical. It will be Γ that will be held constant; we are interested only in the pure effects of changing n, not the secondary cost or production changes that may ensue. We should also note, in passing, that $\Gamma_1 = C_1$, $\Gamma_{12} = C_{12}$, and $n\Gamma_{11} = C_{11}$.

The comparative advantage emerges from these simplifications in the form

$$\Delta'(n) = -(1/2)\{(H_{11} + B_{11}[G(\rho)]^2)/\Gamma_{11}{}^2\}[(1/n)s^2(1 - k) + ks^2]. \quad (15)$$

The number of firms, therefore, has an entirely neutral effect, influencing only the importance of the comparison through the final term of (15). By observing that

$$(\partial/\partial n)[(1/n)s^2(1 - k) + ks^2] = -s^2(1 - k)/n^2,$$

we also see that as long as the firms are not perfectly corrolated ($k \neq 1$), an increase in their number will decrease the absolute magnitude of $\Delta'(n)$, the welfare importance of our comparison. Only when the θ_i are totally independent, however, can the comparison become entirely moot as n increases; that is, only one limit is zero:

$$\lim_{n\to\infty} \Delta'(n)\,|_{k=0} = 0.$$

Perfect correlation ($k = 1$), meanwhile, blunts the effect entirely.

While neutrality may be surprising at first, it is easily explained. There are only two ways that n could influence the choice. The first lies in the potential efficiency gains that are usually afforded price controls because they guarantee that marginal costs will be equal across all firms. In this case, however, the response to θ_i under either mode is given by $\hat{q}_{0i} - [\alpha_i(\theta_i)/C_{11}{}^i]$; quantity standards achieve equal marginal costs, as well. Any potential gains from diversification are similarly destroyed; the output response of each firm to θ_i under quantities, and the corresponding emissions response under prices, are both described by $[x_i(\theta_i)/C_{11}{}^i]$. Variances in total output are therefore never different. Rather than favoring one control over the other, then, changing the number of firms simply effects the importance of worrying about which types should be chosen.

The reader should finally note that the usual superiority of the "gray market" scheme of selling a fixed number of unit pollution permits in lieu of setting specific standards for each firm is also neutralized. Such market schemes are usually thought to be better because they not only fix total emissions, but also allocate those emissions so that the marginal cost of cleaning effluent is equal over the entire industry. We have already noted, however, that both controls automatically achieve this desired efficiency as the firms react to the θ_i.

V. THE CONSUMPTION DISTORTION

As we observed in the introduction, the amount of pollution emitted need not equal the quantity actually "consumed" by the public. A myriad of weather and topographical random variables typically intervene. Returning to the single-firm model, we will call that intervention the consumption distortion, and assume that the amount of pollution emitted (z_e) is additively related to the amount actually consumed (z_c); that is,

$$z_c = z_e + \psi(\lambda),$$

where λ indexes the relevant random variables and ψ prescribes their effect. We also presume that λ is independently distributed.

While the incorporation of this distortion will have a decided impact on the center's optimal control orders, the significant ramifications lies beyond their computation. We therefore simply observe that either a modified quantity standard, $\hat{z}_e'$ or a modified effluent charge, $\tilde{t}'$, is now required. If, as should be expected, $E\psi(\lambda) < 0$, then it can be shown that

$$\hat{z}_e' > \hat{z}_0,$$
$$\tilde{t}' < \tilde{t}$$

and, since λ and θ are independent,

$$E\tilde{z}_e(\tilde{t}', \theta) = \hat{z}_e'.$$

The producer of x, meanwhile, is not directly effected by $\psi(\lambda)$, so that the responses recorded by Eqs. (7) and (11) remain valid. As a result,

$$\tilde{x}'(\tilde{t}) > \tilde{x}(\tilde{t})$$

under prices,

$$\hat{x}'(\theta) > \hat{x}(\theta)$$

under quantities, and

$$E\hat{x}'(\theta) = \tilde{x}'(\tilde{t}').$$

The sole effect of the consumption distortion has been to cause output and emissions to be systematically increased.[12]

If benefits and social costs are truly quadratic, the comparative advantage will emerge from this complication entirely intact. A quadratic function exhibits a constant curvature throughout its domain. Since it is that curvature which predicts the severity of the variation induced losses, the systematic translation which we have just observed will have no effect. The loss will be the same regardless of where along the domain output variation (e.g.) is inserted. Were either function not quadratic, however, a different story would emerge. Translation of a randomly varying output into a more highly curved region of the benefit function would, for example, exaggerate the decline in expected benefits (over the case of certain output). A less curved region would, of course, dampen that decline.

[12] The independence of λ from θ, ξ, and η contributes significantly to the simplicity of these statements. In as much as λ indexes weather variables, however, independence is not very costly.

Returning to our pollution example, we cannot be sure of the direction in which average pollution consumption will move. There are, indeed, cases in which it will remain $\hat{z}_0$. We do know, however, that output will be higher when the consumption distortion is acknowledged. In as much as satiation predicts that benefit functions are less curved in their upper regions, output variation should become less costly. Since only quantity standards allow such variation, the comparative advantage of prices should unambiguously decline.

VI. CONCLUDING REMARKS

While we have produced results that have very familiar interpretations, the particular structure of a pollution control problem does have an impact. Quantity standards create variation in the output of the final good which must be weighed against the variation in pollution emissions allowed by effluent charges. The ability to substitute for the pollutant in the production of the final good is crucial in determining the degree with which random changes in cleansing costs are transformed, under quantity controls, into output variation. There is, meanwhile, no corresponding effect under price controls.

This secondary output effect has another impact when the random relationship between the amount of pollutant emitted and the amount consumed is recognized. That recognition may require that the average production levels of both the pollutant and the final good be altered. While the pollutant may still enter the social cost function in the same region, output of the final good will now enter the benefit function in a new region. The output variation allowed by standards is therefore more (less) harmful if the new region is more (less) highly curved.

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